Voltage Droop Control in Power Flow Solutions

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Presentation Outline

• History: Line Drop Compensation in Power Flow
• Need: Renewable Plants Q-V characteristic at point of interconnection
• Solution: Introduction of Voltage Droop Control (with deadband)
• Extension to Traditional Generator (Voltage setpoints and Tolerances)
• Implementation Details
  – Remaining slides included details regarding how to implement this in software. Presentation and white paper are also available.
History: Line Drop Compensation in Power Flow
Start with Big Thermal Plants

• In traditional power flow solution
  – Control either terminal bus or remote bus
  – (PV bus concept)

• In about 2002, BPA expressed a need to model reality
  – Line drop compensation
  – Often they control part way through the step-up transformer
What was BPA doing before 2002

- Add in some fictitious buses and branches
  - 2 series branches that have a net impedance of zero
  - Regulate the fictitious middle bus
  - This works, but fictitious network elements caused confusion

Regulate this middle fake bus
What PowerWorld Implemented in June 2002 (does not require fake bus)

- Line Drop Compensation for Xcomp value only
  - Replace voltage equation in the power flow solution with equation that enforces voltage at fictitious bus
  - $X > 0$ represents controlling a voltage looking out into the system (Line Drop)
  - $X < 0$ represents controlling a voltage looking backwards (Reactive Current Compensation)
Fields of the Generator Object

• Specify **Use LDC_RCC**
  – **YES**: means use special modeling
  – **NO**: means do not use special modeling
  – **PostCTG**: means only do this during a post-contingency power flow solution

• Enter an impedance value for XLDC_RCC
Renewable Generator Questions started in 2013

• Renewable generators regulate a point closer to the point of interconnection

• From a software standpoint
  – Regulation point is a fixed impedance away from the generator terminal
  – This looks like Line Drop Compensation but....
  – We need a Rcomp too → feeder has a large R/X ratio!
  – Added the Rcomp in PowerWorld in December 2014
Generator Fields

- Use LDC_RCC
- RLDC : new in 2014
- XLDC
- This works on a bus because we know the flow associated with line drop is the generator output

<table>
<thead>
<tr>
<th>Number of Bus</th>
<th>Name of Bus</th>
<th>Nom kV of Bus</th>
<th>ID</th>
<th>Use LDC_RCC</th>
<th>RLDC_RCC</th>
<th>XLDC_RCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10318 My name 1</td>
<td>22.00</td>
<td>1</td>
<td>NO</td>
<td>0.00000</td>
<td>0.00010</td>
</tr>
<tr>
<td>11</td>
<td>10319 My Name 2</td>
<td>24.00</td>
<td>1</td>
<td>YES</td>
<td>0.04500</td>
<td>0.08000</td>
</tr>
<tr>
<td>12</td>
<td>10320 My Name 3</td>
<td>22.00</td>
<td>1</td>
<td>NO</td>
<td>0.00000</td>
<td>0.00010</td>
</tr>
<tr>
<td>13</td>
<td>10321 My Name 4</td>
<td>22.00</td>
<td>1</td>
<td>YES</td>
<td>0.04500</td>
<td>0.08000</td>
</tr>
<tr>
<td>14</td>
<td>10394 My Name 8</td>
<td>18.00</td>
<td>1</td>
<td>NO</td>
<td>0.00000</td>
<td>0.00010</td>
</tr>
</tbody>
</table>
Need: Renewable Plants Q-V characteristic at point of interconnection
Similar questions for renewables continue

• Starting to get questions about Solar PV plant voltage control
  – Also similar for wind farms
• How to implement in the power flow solution
Traditional Generator Power Flow Model

- PV and PQ bus
  - Either meeting the voltage setpoint (PV)
  - Or at a Qmax or Qmin limit (PQ)
Slope Control

• Getting questions about solar farms that have voltage control that is not a setpoint

This can be *approximated* using Rcomp/Xcomp

Not the same because using Rcomp/Xcomp, Power distorts this curve
Slope Control with Deadband

• Getting questions about solar farms that have voltage control that is not a setpoint

• A deadband is given
  – 0.98 to 1.02 per unit voltage – provide zero Mvars (or a constant value)

• Once outside these deadband, a negative slope characteristic is followed

• Maximum and Minimum Mvar will be hit eventually

• The various transient stability models have features like this (REPC_A) → power flow however does not
Reactive Power as Function of Voltage

\[ Q_{\text{max}} \]  
\[ Q_{\text{db}} \]  
\[ Q_{\text{min}} \]  

\[ V_{\text{low}} \quad V_{\text{dblow}} \quad V_{\text{dbhigh}} \quad V_{\text{high}} \]  

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Conclusion in Power Flow

• Existing compensation with an impedance has limitations
  – Will not match Q/V characteristic exactly because MW Power affects the calculation using impedance
  – No deadbands possible
  – Can not coordinate between multiple generators

• Must implement a new software feature to fully model this
  – The following has been completely added to PowerWorld Simulator Version 21 Beta
Solution: Introduction of Voltage Droop Control (with deadband)
Merge this with remote regulation: What extra data is needed?

- What voltage is being controlled?
  - Use the regulated bus specification with generators

- **What reactive power is being used in compensation calculation?**
  - User does **not** need to provide this. Software will look at the topology of the system to figure this out

---

**Diagram:**

- Generators are all configured to regulate the Controlled Bus.
You might have two separate groups of generators that regulate the same bus, but operate on different Droop Curves

– Green and Blue generators in separate groups
– This is OK, because of the Droop! Not a voltage setpoint

Generators are all configured to regulate the RegBus
## PowerWorld Simulator: New Object: VoltageDroopControl

<table>
<thead>
<tr>
<th>Field</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Name</strong></td>
<td>String</td>
<td>Unique Identifier for the Object</td>
</tr>
<tr>
<td><strong>Enabled</strong></td>
<td>Boolean</td>
<td>Indicates if the control is turned on or off for the control</td>
</tr>
<tr>
<td><strong>QAuto</strong></td>
<td>Boolean</td>
<td>NO means to use the Qdb, Qmax, and Qmin values directly. If set to YES, then the values of Qdb, Qmax and Qmin are automatically calculated based on the summation of generator Qmax and Qmin</td>
</tr>
<tr>
<td><strong>VDeviation</strong></td>
<td>Boolean</td>
<td>NO means to use the values of Vlow, Vdblow, Vdbhhigh, and Vhigh directly. If set to YES, then the input values of Vlow, Vdblow, Vdbhhigh, and Vhigh are interpreted as deviations away from the voltage setpoints of the generators</td>
</tr>
<tr>
<td><strong>Qdb</strong></td>
<td>Float</td>
<td>The reactive power in Mvar between $V_{dblow}$ and $V_{dbhhigh}$</td>
</tr>
<tr>
<td><strong>Qmax</strong></td>
<td>Float</td>
<td>The maximum reactive power in Mvar for voltages below $V_{low}$</td>
</tr>
<tr>
<td><strong>Qmin</strong></td>
<td>Float</td>
<td>The minimum reactive power in Mvar for voltages above $V_{high}$</td>
</tr>
<tr>
<td><strong>Vlow</strong></td>
<td>Float</td>
<td>Voltage in per unit below which the reactive power is $Q_{max}$</td>
</tr>
<tr>
<td><strong>Vdblow</strong></td>
<td>Float</td>
<td>Voltage in per unit above which the reactive power is $Q_{db}$</td>
</tr>
<tr>
<td><strong>Vdbhhigh</strong></td>
<td>Float</td>
<td>Voltage in per unit below which the reactive power is $Q_{db}$</td>
</tr>
<tr>
<td><strong>Vhigh</strong></td>
<td>Float</td>
<td>Voltage in per unit above which the reactive power is $Q_{min}$</td>
</tr>
</tbody>
</table>
Droop Curve: Reactive Power as Function of Voltage

\[ Q_{\text{max}} \]
\[ Q_{db} \]
\[ Q_{\text{min}} \]

\[ V_{\text{low}} \quad V_{db\text{low}} \quad V_{db\text{high}} \quad V_{\text{high}} \]

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Droop Curve:
Reactive Power as Function of Voltage

• When $Q_{\text{auto}}=\text{YES}$, this means
  
  $Q_{\text{maxUsed}} = \text{Summation of Generator MvarMax}$
  $Q_{\text{minUsed}} = \text{Summation of Generator MvarMin}$
  $Q_{\text{dbUsed}} = 0$

• When $V_{\text{deviation}}=\text{YES}$, this means
  
  $V_{\text{set}} = \text{Voltage setpoint from generators}$
  $V_{\text{lowUsed}} = V_{\text{set}} + V_{\text{low}}$
  $V_{\text{dblowUsed}} = V_{\text{set}} + V_{\text{dblow}}$
  $V_{\text{dbhhighUsed}} = V_{\text{set}} + V_{\text{dbhhigh}}$
  $V_{\text{highUsed}} = V_{\text{set}} + V_{\text{high}}$

• White paper spells out special cases as well
Comment about “Qmax” and “Qmin”

- For renewable energy plants, “Plant Qmax” and “Plant Qmin” is not always equal the summation of the individual generator MvarMax and MvarMin
  - Thus we enforce the curve as shown regardless
- When using the flag \texttt{Qauto=YES} however, we follow the slope base the Vhigh and Vlow points
Droop Curve:
Reactive Power as Function of Voltage

\[ Q_{\text{db}} \]

\[ Q_{\text{max}} \]

\[ Q_{\text{min}} \]

\[ V_{\text{low}} \]

\[ V_{\text{dblow}} \]

\[ V_{\text{dbhhigh}} \]

\[ V_{\text{high}} \]

\[ Q_{\text{auto}}=\text{YES} \]

\[ =\text{NO} \]

\[ \text{NO}=Q_{\text{auto}} \]

\[ \text{YES}=Q_{\text{auto}} \]
New Field for a Generator: **VoltageDroopControl**

• **VoltageDroopControl** is a string field for a generator
  – This references the name of the **VoltageDroopControl** object to which the generator belongs
  – A blank string indicates it does not belong to such a control (default is blank)
Things that are NOT extra inputs

• The following are automatically determined by the software
  – The regulated bus (RegBus) of the \textit{VoltageDroopControl} is determined by looking at the RegBus of the generators that are assigned to the \textit{VoltageDroopControl} object
    • Also any buses connected by very low impedance branches are considered the same RegBus by the software
    • This uses the “ZBR Threshold” which defaults to 0.0002 in PowerWorld Simulator
  – A list of “Arriving Branches” will be automatically determined by looking at the network which connects the generators in the \textit{VoltageDroopControl} to the RegBus
Example

• 2 VoltageDroopControls: “Droop A”, “Droop B”

• Several generators assigned to these
Voltage Droop Control Dialog
Coordination Between Generators

• The implementation in the power flow solution is similar to the implementation for coordination of remotely regulating generators that maintain a voltage setpoint (PV bus concept)
  – Same concept is used for sharing Mvar between multiple generators assigned to same VoltageDroopControl and Regulated Bus

• Then just replace the voltage equation with an equation representing the voltage droop curve
  – Multiple VoltageDroopControls sharing a regulated bus adds some complexity though
• Define the following
  – **MvarArriving** = Summation of Mvar on AC branches arriving at Regulated Bus coming from the generators
  – **MvarGenRegBus** = Summation of Mvar for Generators assigned to the Voltage Droop Control that are at the regulated bus
    • Note: This includes generator with AVR=NO.
    • Any generator in the VoltageDroopControl contributes to the Mvars regardless
  – **MvarDroopCurve** = Evaluation of the Droop Characteristic Curve at the per unit voltage at the regulated Bus

• **VoltageDroopControl** will enforce an equation that

\[
\text{MvarDroopCurve} = \text{MvarArriving} + \text{MvarGenRegBus}
\]

– This replaces the voltage equality that is used in remotely regulated *voltage* equations that have always existed
Example Operating Point

Droop A = -27.96 Mvar
Droop B = -14.97 Mvar = Summation

Mvars are proportional to Remote Reg %

Light Blue ➔ Mvar for generators OFF AVR
Dark Blue ➔ Mvar for generators ON AVR
Example Vreg = 1.028979

\[ Q = 10 + (-50 - 10) \frac{(1.028979 - 1.02)}{(1.05 - 1.02)} = -27.958 \]
Droop B:
Example Vreg=1.028979

• Example Operating Point For “Droop B”

\[ Q = 0 + (0 - 50) \frac{(1.028979 - 1.02)}{(1.05 - 1.02)} = -14.965 \]
PowerWorld shows multiple characteristics at Regulated Bus

Choose a Regulated Bus and it super-imposes all Voltage Droop Characteristics at this bus
Generator Limits: Same as before

- Handling individual generator limits must still be handled
  - Same as existing remotely regulating voltage control
  - Also need to handle that there can be generators at the regulated bus that belong to the same VoltageDroopControl

\[
\begin{align*}
Q_v(V_{\text{reg}}) & \quad Q_{\text{max}} \\
Q_{db} & \quad Q_{\text{min}} \\
V_{\text{low}} & \quad V_{db\text{low}} & \quad V_{db\text{high}} & \quad V_{\text{high}}
\end{align*}
\]
Generators Hitting Limits

• Change voltage setpoint at bus 2 to 1.05 per unit
• Generators at bus 1, 9, and 11 all hit Mvar Limits
• All generators in “Droop B” are at limits

1.033622 pu
Droop B at Mvar limits

- Droop B no longer operating on the Curve

Mismatch = -3.65
Closeup of Red Dot not on Curve
Extension to Traditional Generator (Voltage setpoints and Tolerances)
Beyond just Renewable Plants: Generator Voltage Control

• How is generator voltage control modeled now
  – Voltage Setpoint and then Qmax and Qmin

  – Example
    • Vset = 1.01
    • Qmax = +500
    • Qmin = −400
What are instructions in real life given to a generator operator?

• Generators are given a setpoint (1.035 per unit)
  But they are also given a tolerance!
  – I believe typical values are 0.25% to 2.0%
  – Example: 0.5% = 0.005 per unit, so voltage is really instructed to be between 1.030 and 1.040

• This can be beneficial to consider in to context of a power flow solution!
  – Will give the power flow solution some more flexibility of implemented appropriately
As an example, consider situation with
- \((Q_{\text{min}}, Q_{\text{max}}) = (-400 \text{ Mvar}, +500 \text{ Mvar})\)
- \(V_{\text{set}} = 1.000\), Tolerance = 0.01 (1%)

Thus the guidance for a generator is really
- **Stay in the yellow box!**
- Anywhere is fine
Stay in the Yellow Box!

• In a Power Flow solution, need a more precise statement than just “stay in the yellow box”
  – Otherwise there is no unique solution to the power flow and the initial condition will effect the results
  – For the purposes of a power flow solution, we need
    • $\frac{dQ}{dV}$ must be negative across entire range
      – We can discuss off-line why this is important
    • Must match at the upper left and lower right corners of our box ($Q$ is a continuous function of $V$)
Use Voltage Droop Control Concept

• Add the two green lines below
  – Draw first line (Vlow, Qmax) to (Vset, 0.0)
  – Draw second line from (Vset, 0.0) to (Vhigh, Qmin)

  • Qmax = +500
  • Qmin = −400
  • Vlow = 0.99
  • Vdblow = 1.00
  • Vdbhhigh = 1.00
  • Vhigh = 1.01
Software Implementation

- Define **ONE** VoltageDroopControl
- Assign 100s or 1000s of generators to do this!

QAuto = YES
Vdeviation = YES
Vlow = -0.010
Vdblow = -0.000
Vdbhight = +0.000
Vhigh = +0.010
Software Implementation

• Software automatically assigned an QV characteristic equation to each unique regulated bus inside the VoltageDroopControl
  – It will group generators that regulate the same bus (or group of buses connect by small impedances)
• Allows you to quickly assign deadband and droop to 1000s of generators quickly.
Conclusion

• PowerWorld Simulator Version 21 Beta has implemented these new features
  – New Object: VoltageDroopControl
    • Has a Name and a QV Characteristic curve
  – New Field for Generator
    • Name of VoltageDroopControl

• The remaining slides just have more detail on how this is implemented and what valid network topologies are
Implementation Details
Some network topologies are not allowed

- Separate VoltageDroopControl objects can *not* have overlapping networks to connect to the regulated bus.
- All generators in the VoltageDroopControl must regulate the same bus (or buses connected by zero-impedance branches).
- Having separate VoltageDroopControl objects that share the same regulated bus is fine and expected.
  - For example, you can also have multiple generators at the same bus belong to different VoltageDroopControl objects.
Generator Behavior when VoltageDroopControl is not valid

• If the VoltageDroopControl is blank, or if the VoltageDroopControl.Enabled = NO, then the generator will go back to its normal voltage setpoint behavior

• If the VoltageDroopControl and network topology around the generators is not a valid configuration for control, then the generators in the VoltageDroopControl will act as fixed Mvar outputs
Valid Example

• Two **VoltageDroopControl** objects (Green and Blue)
• Generators assigned to these
• All generators regulate RegBus
• Software automatically determines “Arriving Branches”
  – Qla1 and Qla2 for Group A
  – Qlb1 for Group B
Invalid Topology: Conflicting Control

- The branch highlighted in red below makes this topology invalid (overlapping network)
  - All generators in BOTH of these \textbf{VoltageDroopControls} will be treated as fixed Mvar output because this is invalid

\begin{itemize}
  \item Generators are all configured to regulate the RegBus
\end{itemize}
Invalid Topology: Conflicting Control

• Generator X below is set to regulate voltage in the normal manner
  – This conflicts with control for generators A1, A2, and A3, so VoltageDroopControl A is invalid
  – Generators located at X are OK if they are not on any voltage control (AVR = NO)
Invalid Topology: Can not reach Reg Bus

• Generator A2 is not able to reach the RegBus through the network topology
  – Entire VoltageDroopControl is ignored
  – (If A2 is offline that’s fine, but if A2 is online it must be able to reach the RegBus)
Valid Example

- Assume that Bus 1, 2, 3, 4, and 5 are all connected by branches below the ZBRThreshold, then the following is acceptable input data
  - A2 RegBus = Bus 3
  - A1 RegBus = Bus 2
  - A3 RegBus = Bus 5

Generators are all configured to regulate the RegBus

Treated as a single point by the validation routines
Extreme Example
But this is still Valid Network Topology

- This is important for full-topology models including breakers
- Blue, Orange, and Green represent different VoltageDroopControls
  - Each generator regulates any bus highlighted in Red within the “RegBus”
- White not assigned, but AVR = NO
New Numerical Implementation Issues due to Q versus V Characteristic

• Input Data Issue
  – The slopes between (V_{low} \rightarrow V_{dblow}) and (V_{dbhhigh} \rightarrow V_{high}) may become too large

• QV Characteristic Issue
  – The “corner points” introduce discontinuous derivatives \rightarrow This is very bad for solution algorithms
Define tolerances

- \( V_{tol} = 0.001 \) (this is hard-coded in PowerWorld Simulator)
  - Would not allow \( V_{dblow} = 0.9500 \) and \( V_{low} = 0.9485 \)
- \( \text{MaxSlope} = 1/(0.0002 \times S_{Base}) = 500,000 \text{ Mvar/Vpu} \)
  - This slope acts similarly to a line impedance in the equations, so it is similar to a minimum line impedance, jumper threshold, etc.
  - There is a “ZBR Threshold” in Simulator that defaults to 0.0002
    - User can change this. Other software tools have similar thresholds
  - Example assuming a \( (V_{dblow} - V_{low}) = V_{tol} = 0.001 \)
    - \( Q_{max} = 500.0 \text{ Mvar} \)
    - \( Q_{db} = 0.0 \text{ Mvar} \)
    - \( V_{dblow} = 0.950 \text{ per unit} \)
    - \( V_{low} = 0.949 \text{ per unit} \)
    - \( \text{Slope} = (Q_{max} - Q_{db})/(V_{dblow} - V_{low}) = (500-0)/(0.950-0.949) = 500,000 \)
    - \( \text{MaxSlope} \) only enforced for extreme input data.
    - This means a range of 500 Mvar will force voltage difference of at least 0.001 per unit
Input Data:
Avoiding Numerical Problems

- Modify the input parameters using the following:
  - If $V_{dbhhigh}$ and $V_{dblow}$ are within $V_{tol}$ of one another, set them equal to their average.
  - Force $V_{low}$ to be at least $V_{tol}$ smaller than $V_{dblow}$.
  - Force $V_{high}$ to be at least $V_{tol}$ larger than $V_{dbhhigh}$.
  - Force $Q_{max}$ to not be smaller than $Q_{db}$.
  - Force $Q_{min}$ to not be larger than $Q_{db}$.
  - If slope from $V_{dblow}$ to $V_{low}$ is larger than $MaxSlope$, modify $V_{low}$ so that the slope is equal to $MaxSlope$.
  - If slope from $V_{dbhhigh}$ to $V_{high}$ is larger than $MaxSlope$, modify $V_{high}$ so that the slope is equal to $MaxSlope$. 
• Calculate the “used” values for the QV characteristic

```pseudo-code
// Enforce voltage thresholds that are not too close to one another
If (Vdbhigh - Vdblow) < VTol then
    VdbhighUsed = (Vdbhigh + Vdblow)/2
    VdblowUsed = (Vdbhigh + Vdblow)/2
Else
    VdbhighUsed = Vdbhigh
    VdblowUsed = Vdblow
EndIf
If (VdblowUsed - Vlow) < VTol then VlowUsed = VdblowUsed - VTol
Else VlowUsed = Vlow
If (Vhigh - VdbhighUsed) < VTol then VhighUsed = VdbhighUsed + VTol
Else VhighUsed = Vhigh
// Enforce decreasing relationship between reactive powers
If Qmax < Qdb then QmaxUsed = Qdb
Else QmaxUsed = Qmax
If Qmin > Qdb then QminUsed = Qdb
Else QminUsed = Qmin
// Enforce dQ/dV slopes not too big
MaxSlope = 1/(ZBRThreshold*SBase)
if (QmaxUsed - Qdb)/(VdbLowUsed - VlowUsed) > MaxSlope then
    VlowUsed = VdbLowUsed - 1/MaxSlope*(QmaxUsed - Qdb)
if (QminUsed - Qdb)/(VdbhighUsed - VhighUsed) > MaxSlope then
    VhighUsed = VdbhighUsed - 1/MaxSlope*(QminUsed - Qdb)
```
We have modified input data
Vlow changed to 1.0274 from 1.028

\[
\frac{800 - 0}{1.029 - 1.0274} = \frac{1}{0.0002 \times 100}
\]
Avoiding Numerical Problems due to “corner points”

- Use splines to smooth the corners
- The curve will become the following
Derivatives after smoothing the corners

\[
\frac{dQ_v}{dV_{reg}}
\]

\[
\frac{Q_{max} - Q_{db}}{V_{low} - V_{dblew}}
\]

\[
\frac{Q_{min} - Q_{db}}{V_{high} - V_{dblew}}
\]

\[V_{low}\quad V_{dblew}\quad V_{dblew}\quad V_{high}\]
Avoiding Numerical Problems due to “corner points”

- Use elliptical splines to smooth the corners
  - Curves will follow an ellipse around corners instead
  - May also use cubic function as well
- The curve will become the following

\[ Q_v(V_{\text{reg}}) \]

\[ Q_{\text{max}} \]

\[ Q_{\text{db}} \]

\[ Q_{\text{min}} \]

\[ V_{\text{low}} \quad V_{\text{dblow}} \quad V_{\text{dbhigh}} \quad V_{\text{high}} \]

Smooth
Example QV Characteristic

• Faint Green Line shows the spline
Close-up of Corner Point

Vs = 0.001
Qs = 10 Mvar

Largest Error = 2.17%
Implementation of smoothing

- We start with two line segments from our QV characteristic defined by point \( A \), \( 0 \) and \( B \) below
- Smooth between point \( 1 \) and \( 2 \)
- Scaling is chosen in a specific way to smooth enough that the solution can be achieved, but not more than necessary

\[
(V_a, Q_a) = (V_0 - dV_a, Q_0 + dQ_a)
\]

\[
(V_0, Q_0)
\]

\[
(V_1, Q_1)
\]

\[
(V_2, Q_2)
\]

\[
(V_0 + dV_b, Q_0 - dQ_b)
\]
White Paper

• Details of Spline Function Choice and Implementation in Power Flow solution are included in PowerWorld Corporation’s public website

• Any other software vendor can implement this into their power flow algorithm using this information
Circular Spline Function

\[ Q_{pu} = Q_{scale} \left[ y_c + \text{Sign} \sqrt{R^2 - \left( \frac{V_{pu}}{V_{scale}} - x_c \right)^2} \right] \]

\[ \frac{dQ_{pu}}{dV_{pu}} = \frac{Q_{scale}}{V_{scale}} \left[ \frac{\text{Sign} \left( \frac{V_{pu}}{V_{scale}} - x_c \right)}{\sqrt{R^2 - \left( \frac{V_{pu}}{V_{scale}} - x_c \right)^2}} \right] \]
Cubic Spline Function

\[Q_{pu} = Q_{scale} \left[a + b \left(\frac{V_{pu}}{V_{scale}} - x_{shift}\right) + c \left(\frac{V_{pu}}{V_{scale}} - x_{shift}\right)^2 + d \left(\frac{V_{pu}}{V_{scale}} - x_{shift}\right)^3\right]\]

\[\frac{dQ_{pu}}{dV_{pu}} = \frac{Q_{scale}}{V_{scale}} \left[b + 2c \left(\frac{V_{pu}}{V_{scale}} - x_{shift}\right) + 3d \left(\frac{V_{pu}}{V_{scale}} - x_{shift}\right)^2\right]\]
Advantage of Spline

- Hardest part of existing power flow algorithms
  - Figuring out which generators are at Mvar limits
  - Introduces “equation switching” (similar to PV/PQ bus)
- Without the spline function, the power flow would get a lot more complicated *(and slower)*
  - We would keep track of jumping between each segment of the QV characteristic
  - This is also “equation switching”