Three-Phase Induction Motors

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Load Modeling Update
WECC MVS Meeting Online

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Summary

- Provide a software history of 3-phase induction motor models
- CIM5, CIM6, CIMW, MOTOR1 before 1960
  – The 3-phase induction motor model
- Missing steady-state frequency-dependence of induction motor model CIM5
  – Problem noticed in 2019 by Kannan at ISO-NE
- Side Track: MOTORW – 1997
  – Important step in creating load components
  – Double-cage model problem will be discussed

3-phase induction motor model. The real model

- Stator containing armature windings
- Rotor windings are shorted
- Typically have 2 rotor windings (“double cage”)
  - “Cage” ultimately means a winding

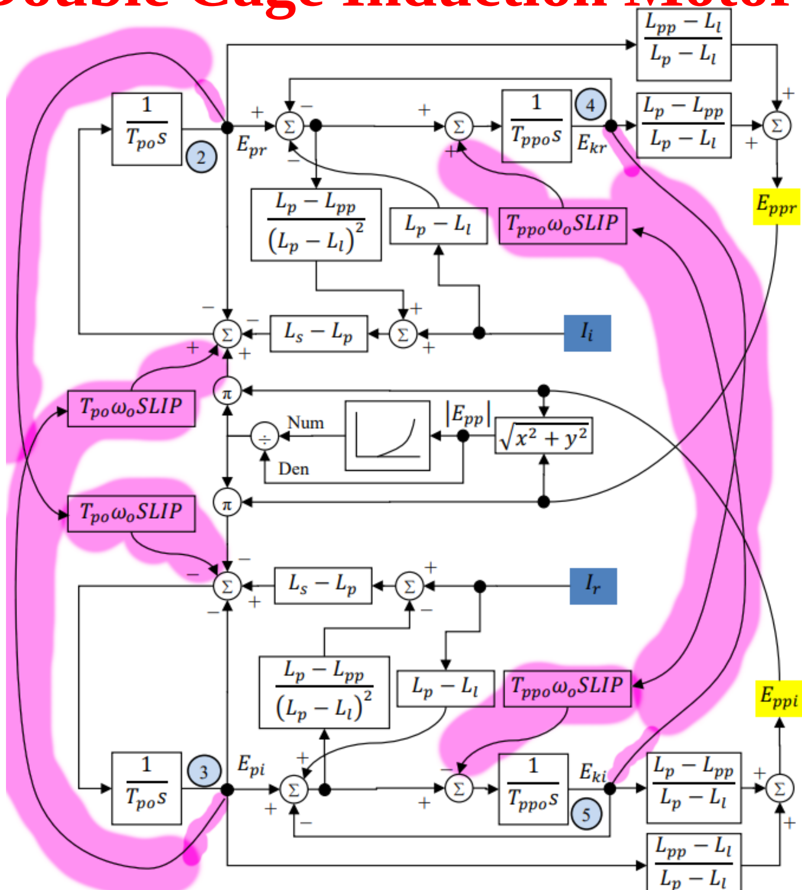
- The mathematical theory of a 3-phase induction motor is the same as a 3-phase synchronous machine
  - Synchronous machine is simplified because it always operates very close to synchronous speed

Induction Motor Model: CIM5 What's different?

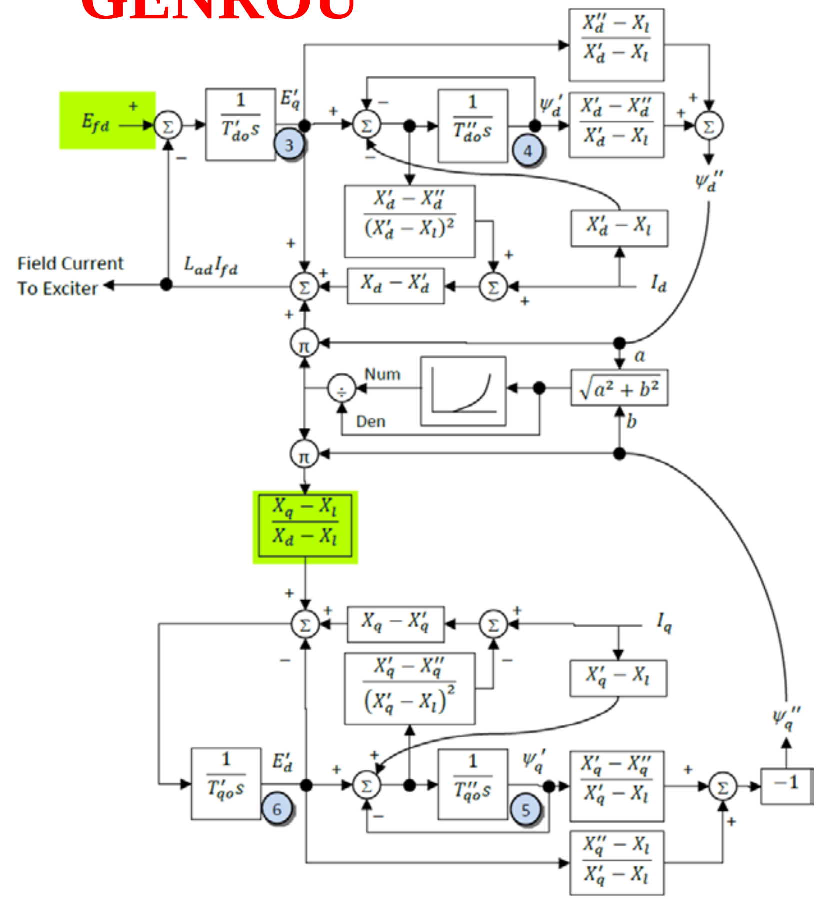
- Some sign differences because current conventions (motor vs. generator convention)
- Terms for "SLIP" create a feedback between the flux dynamic states
  - Note: SLIP is not the normal definition of slip, but similar
- There is no input field voltage
  - Rotor windings are shorted
- Otherwise, these models are very similar
  - 2 windings each have a complex flux, so that's 4 states variables – same concept as GENROU
  - Speed state is modeled, with mechanical torque a function of rotor speed optionally
  - GENROU had a rotor angle which isn't a state variable for induction machine

Induction Motor and Synchronous Machine are very similar!

Double Cage Induction Motor



GENROU



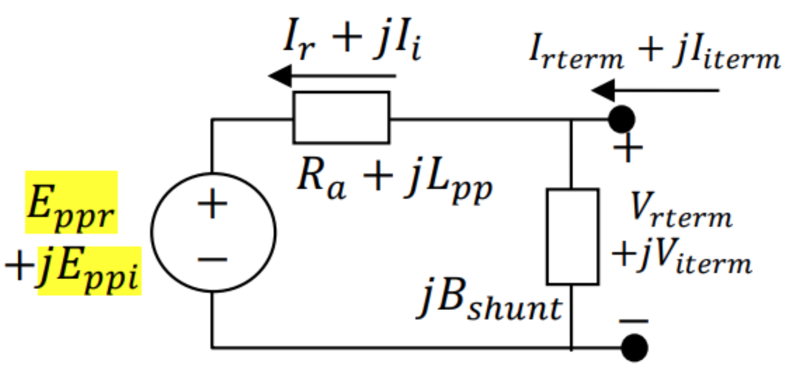
No Efd on Induction Motor

SLIP Terms do not exist for synchronous machine

Network Interfaces

CIM5, MOTOR1
Double Cage Induction Motor
Equation in Network Reference

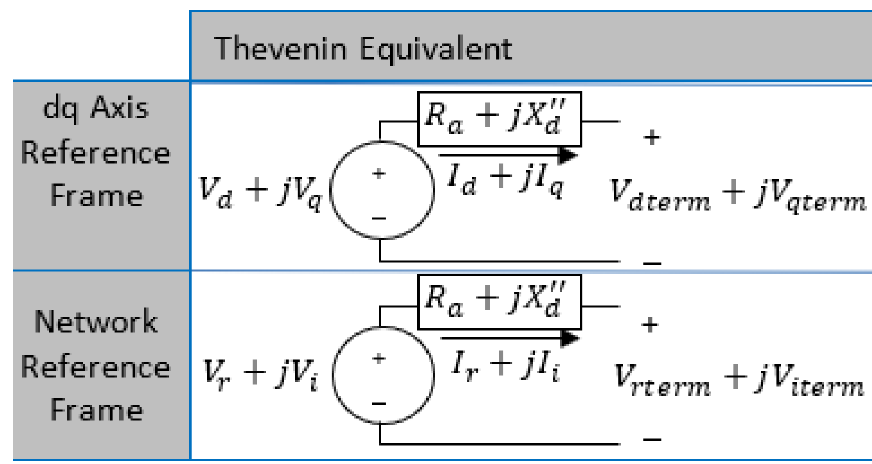
$$V_{source} = \left(E_{ppr} + jE_{ppi}\right)$$



B_{shunt} is determined as part of model initialization

GENROU
Both a dq rotor reference
And a network reference

$$V_d + jV_q = j(1+\omega)\left(\psi_d'' + j\psi_q''\right)$$



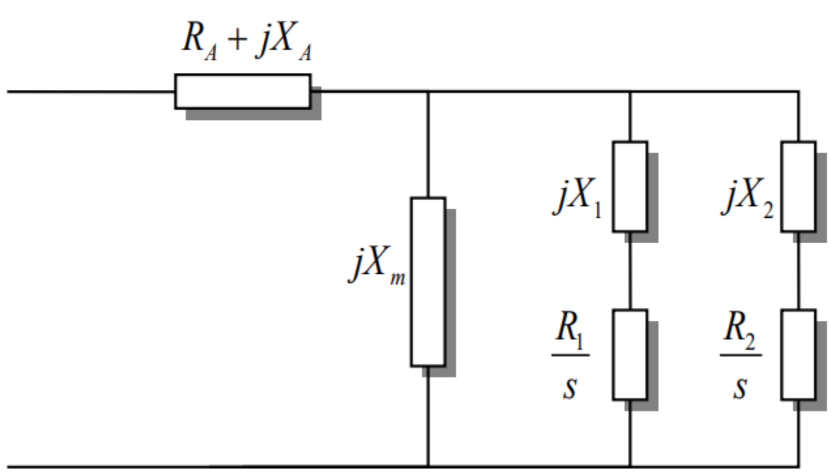
Rotor Angle provides relationship between dq and network reference frame

3-phase Induction Motor Model History

- CIM5 is the load motor model from textbooks from well before the 1960s
  - This is a PSS/E model and has variations named CIM6 and CIMW which make modifications to the mechanical load connects to it
  - 7 Circuit parameters
    - Ra, Xa, Xm, R1, X1, R2, X2
- MOTOR1 is a generator model in PSLF which uses the same set of equations
  - 7 Dynamic parameters
    - Ls, Lp, Ra, Tpo, Lpp, Tppo, Ll

Circuit Parameters vs Dynamic Parameters

- 7 parameters
  - Algebraic relationship between parameter sets
  - Doesn't matter which one you use, but I prefer circuit parameters



	Double Cage	Single Cage
R_a	R_a	R_a
L_s	$Xa + Xm$	$Xa + Xm$
L_l	Xa	Xa
L_p	$Xa + \frac{1}{\frac{1}{Xm} + \frac{1}{X1}}$	$Xa + \frac{1}{\frac{1}{Xm} + \frac{1}{X1}}$
L_{pp}	$Xa + \frac{1}{\frac{1}{Xm} + \frac{1}{X1} + \frac{1}{X2}}$	$Same\ as\ L_p$
T_{po}	$\frac{X1 + Xm}{\omega_o R1}$	$\frac{X1 + Xm}{\omega_o R1}$
T_{ppo}	$\frac{X2 + \frac{X1 * Xm}{X1 + Xm}}{\omega_o R2}$	0

Mechanical Torque Equation

- CIM5: $\text{Load Torque} = T(1+\Delta\omega)^D$
- CIM6: $\text{Load Torque} = T(A\omega^2 + B\omega + C0 + D\omega^E)$
- Composite load model uses same as CIM5
  - Parameter we use is normally "Etrq" for "exponent of torque equation"
- Mechanical Torque Equation impacts whether a motor will restart itself after voltage recovers
  - Constant Torque (Etrq = 0) makes it harder to restart
  - Higher exponent (Etrq = 2) makes is easier for motor to restart

Adding Frequency Dependence to Algebraic Models

- Algebraic Models that add frequency dependence
- LDFR – old PSS/E model
  - Provides a very simple algebraic model which represents the frequency dependence
  - Applies to constant power and current only

$$P = P_O\left(\frac{\omega}{\omega_O}\right)^m \qquad Q = Q_O\left(\frac{\omega}{\omega_O}\right)^n$$

$$I_p = I_{po}\left(\frac{\omega}{\omega_O}\right)^r \qquad I_q = I_{qo}\left(\frac{\omega}{\omega_O}\right)^s$$

- WSCC Model – old PSLF Model

$$P = P_o[p_1V^2 + p_2V + p_3][1 + lpd(freqpu - 1)]$$
$$Q = Q_o[q_1V^2 + q_2V + q_3][1 + lpq(freqpu - 1)]$$

- IEEL Model – old PSS/E Model

$$P = P_{load}\left(a_1v^{n_1} + a_2v^{n_2} + a_3v^{n_3}\right)\left(1 + a_7\Delta f\right)$$

$$Q = Q_{load}\left(a_4v^{n_4} + a_5v^{n_5} + a_6v^{n_6}\right)\left(1 + a_8\Delta f\right)$$

3-phase Induction Motor, Stator Frequency Effect

- In 2019, Dmitry Kosterev of the Bonneville Power Administration (BPA) was working with Kannan Sreenivasachar from ISO-New England (ISONE)
  - Kannan was noticing that as he decreased the frequency in the system, the steady state electric power of induction motor load was not impacted by frequency changes
- As a new steady-state is reached, a constant *torque* mechanical load results in a constant *power* electrical load
  - Thus electrical frequency drop does not impact electrical power at steady state.
- Dmitry emailed me to ask why this was occurring
  - My response initially was “that’s what is supposed to happen”
    - I had noticed this in the equations coded for a three-phase induction motor 10 years ago, and just assumed it was fine
  - BPA, ISONE, and EPRI have experience testing hardware, and to all of them this did not seem right
  - They tested in 4 different software tools and all showed the same response
  - Time to revisit induction machine theory!

Revisit network interfaces Induction and Synchronous Machines

Double Cage Induction Motor

Equation in Network Reference

GENROU

Both a dq rotor reference

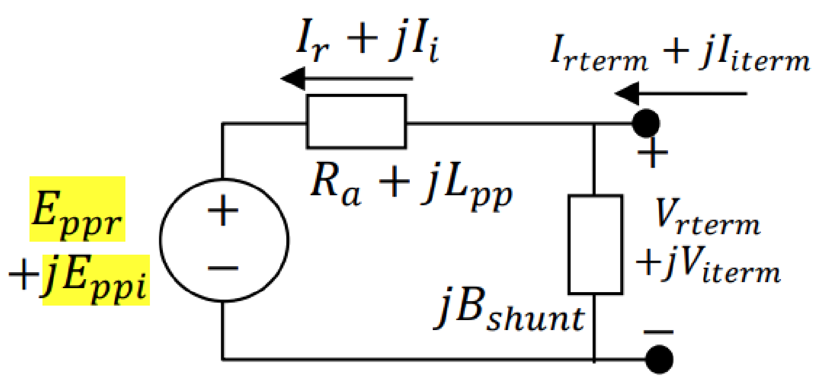
And a network reference

Multiply by frequency was not there!

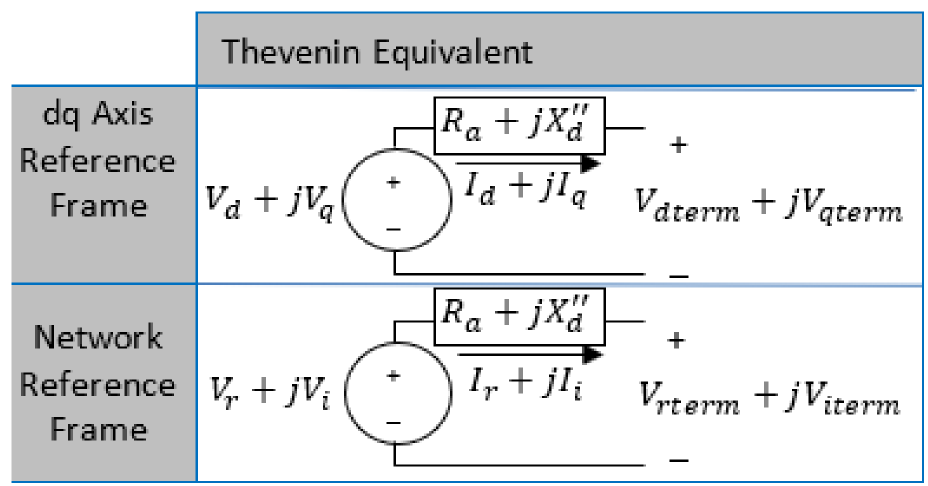
This is what is being fixed

$$V_{source} = \boxed{}\left(E_{ppr} + jE_{ppi}\right)$$

$$V_d + jV_q = j\boxed{(1+\omega)}\left(\psi_d'' + j\psi_q''\right)$$



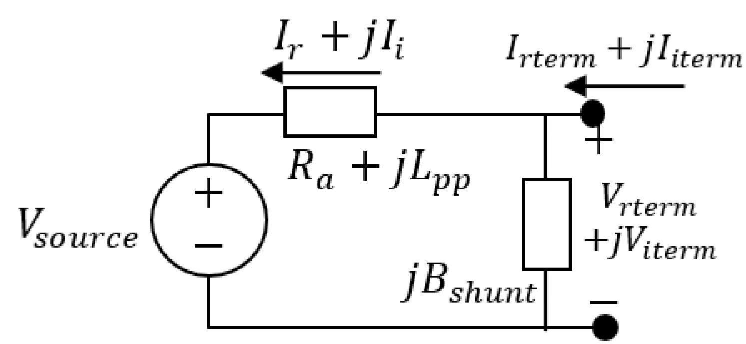
B_{shunt} is determined as part of model initialization



Network Interface for Models

- Differential equations give output of *fluxes*
- The network interface equation is a voltage relationship, so we should multiply by the *stator electrical frequency* in per unit ω_{bus}
- This term was missing in traditional software implementation

$$V_{source} = \omega_{bus}(E_{ppr} + jE_{ppi})$$



B_{shunt} is determined as part of model initialization

Reminder: Why do we multiply flux by ω to get voltage

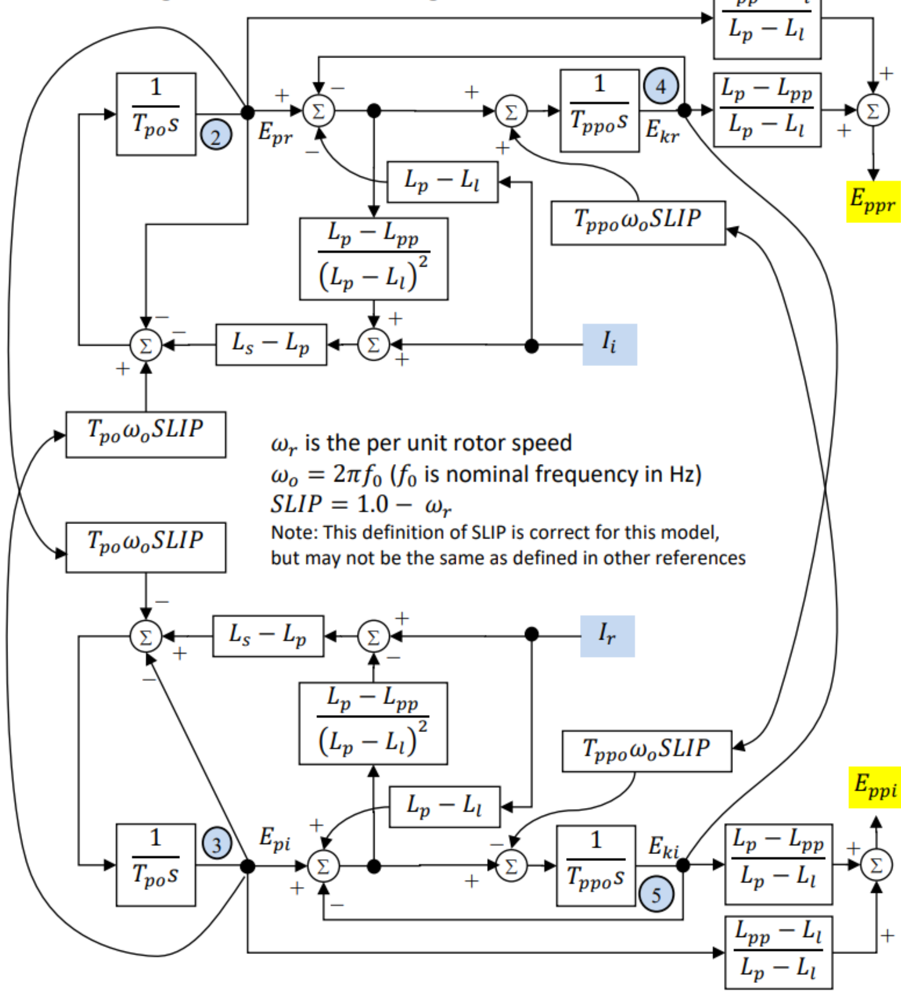
- Remember these things are all phasors which represent time-varying waveform at a frequency
- $\psi = \psi_{mag}[\cos(\omega t - \theta) + j\, sin(\omega t - \theta)]$
- $\psi = \psi_{mag} e^{j(\omega t - \theta)}$
- $V = \frac{d\psi}{dt} = \psi_{mag}\left[e^{j(\omega t - \theta)}\right] j\omega$
- $V = j\omega\, \psi$
- Voltage = Flux *multiplied by ω* and also apply a 90 degree phase shift (multiply by *j*)

This fixes our problem

- With multiplication added, a constant *torque* load will no longer behave as a constant electric *power* load at the network level
- PowerWorld Simulator has added a new model named **InductionMotor3P_A**
- https://www.powerworld.com/WebHelp/#TransientModels_HTML/Load%20Characteristic%20InductionMotor3P_A.htm

InductionMotor3P_A

Double-Cage Motor Block Diagram



Mechanical Equation

$T_{elec} = E_{ppr}I_r + E_{ppi}I_i$

(1) $\frac{d\omega_r}{dt} = \frac{1}{2H}\left(T_{elec} - T_{nom}\omega_r{}^{Etrq}\right)$

Network Interface Equations

$StatorCurrent = I_r + jI_i$

$Z_{source} = R_s + jL_{pp}$

$Y_{source} = \frac{1}{R_s + jL_{pp}} = G + jB$

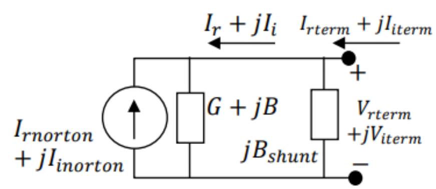
ω_{bus}= terminal bus frequency in per unit

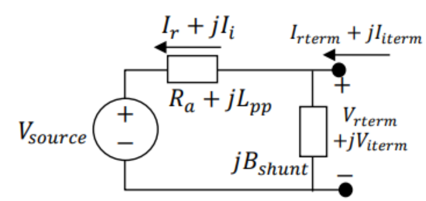
If $flag \leq 0$

Then $V_{source} = (E_{ppr} + jE_{ppi})$

Else $V_{source} = \omega_{bus}(E_{ppr} + jE_{ppi})$

$I_{rnorton} + jI_{inorton} = V_{source}(G + jB)$





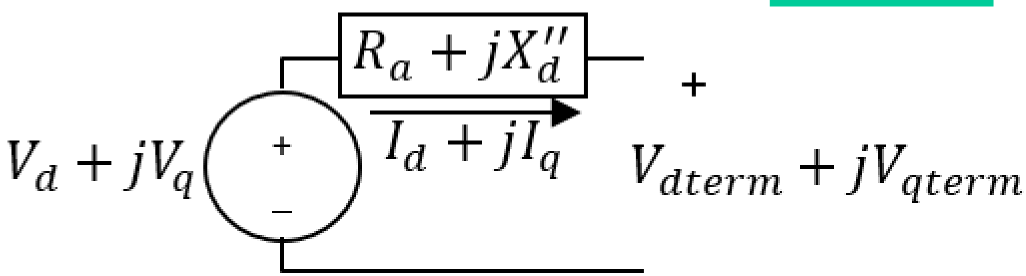
B_{shunt} is determined as part of model initialization

Flag parameter indicates whether to include this multiplication

Similar to Synchronous Machine

- Same concept is in the synchronous machine models too
- GENROU and related models have multiplied by **<u>rotor speed</u>** instead
    - ω = rotor speed deviation in per unit

$$V_d + jV_q = \left(-\psi''_q + j\psi''_d\right)(1+\omega)$$



- Should we be using the stator electrical frequency for a synchronous machine too?
    – We don't need to because $\omega_r \rightarrow \omega_e$ in synchronous machine this approximation is fine for synchronous machine

Comparing Induction Machine and Synchronous Machine

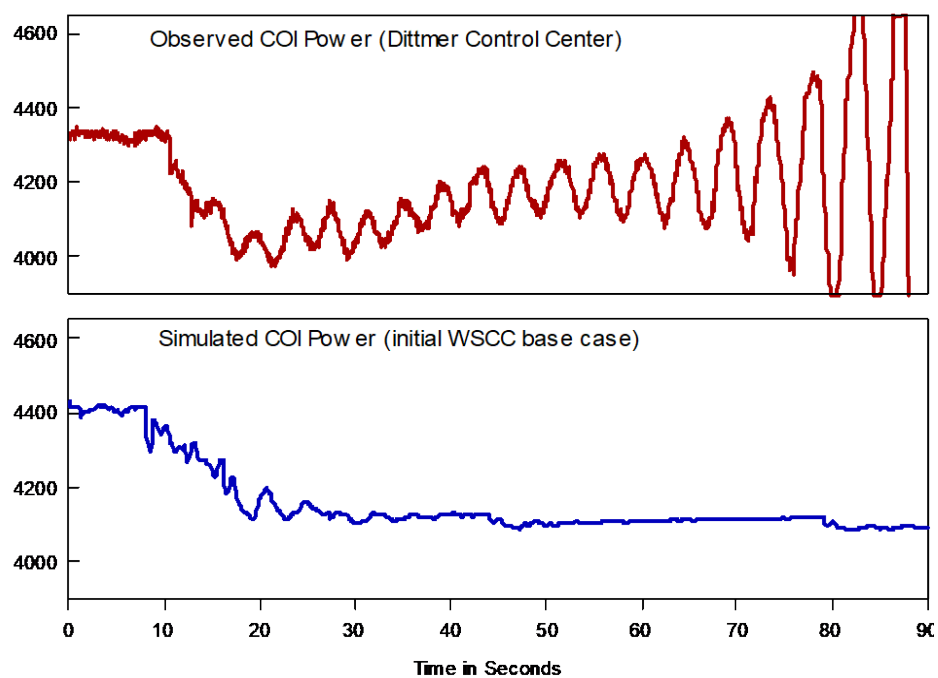
	Synchronous Machine	**Induction Machine**
Transient Time Frame	Small deviations in rotor speed during simulation	Large deviations in rotor speed during simulation
Steady State	Rotor speed and stator electric speed are equal at steady state	Rotor speed is not equal to electric speed at steady state (always a slip)
Is using Rotor Speed instead of stator electric speed a good approximation?	YES, Good approximation	NO, Must use stator frequency in per unit

Side Track: MOTORW

- The PSLF model MOTORW is important in the history of including induction motor models
  - Was adding after 1996 blackout in WECC

Oscillations in real life

Did NOT match numeric simulations

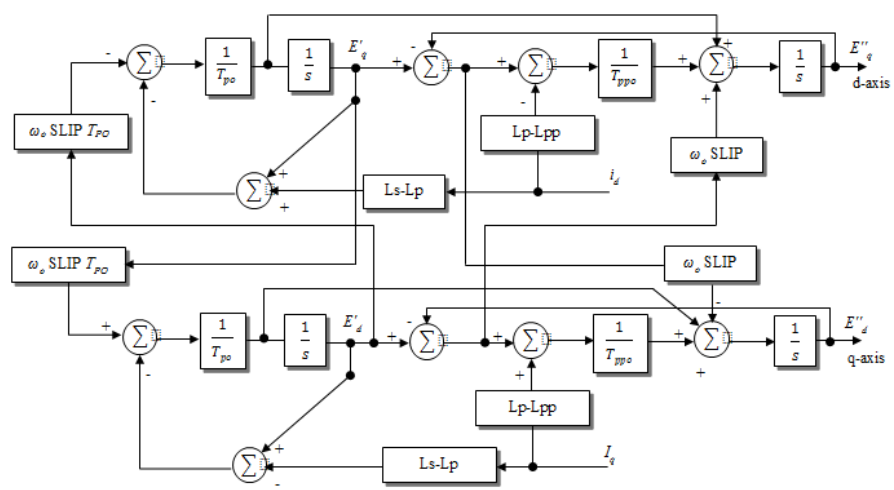


Side Track: MOTORW

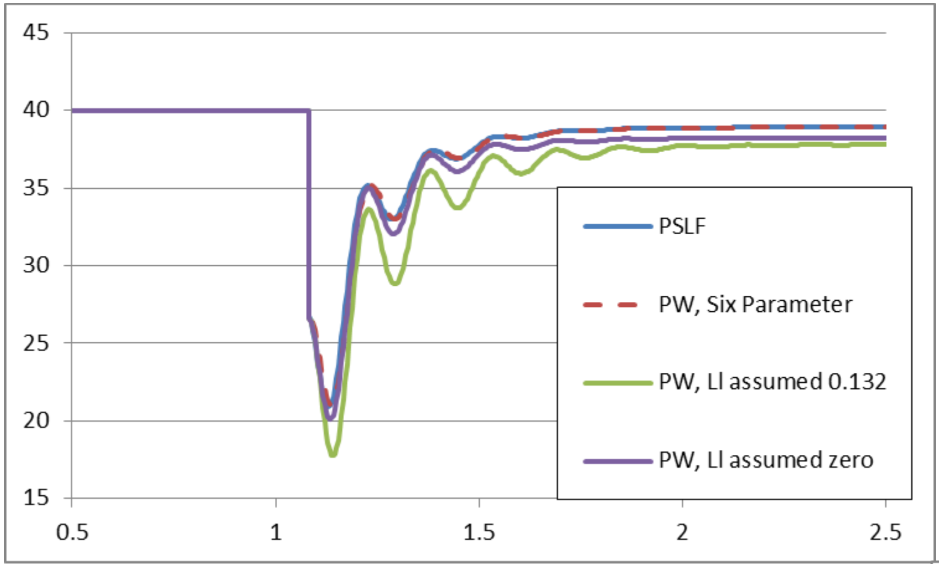
- Provided a first step in modeling a “partial” load characteristic
- MOTORW: had a parameter indicating a fraction
  - “Load is 20% 3-phase induction motor”
  - New concept which started us down the path of the composite load model
- MOTORW was called the “interim load model” from about 1997 – 2016, so it had a long run!
- For a single-cage motor [(Tppo = 0) or (Lpp=Lp)] MOTORW and CIM5 simply to same equations
  - Historically MOTORW models had input data that represented single-cage motors only
  - Old WECC MOTORW model always used Tppo = 0 and Lpp=Lp=0.17

Side Track: MOTORW Double Cage Model

- However…. MOTORW double-cage equations are not the same as CIM5
- 6 parameters: Ls, Lp, Ra, Tpo, Lpp, Tppo
  - It is missing the leakage reactance (Ll)
  - Model however is just ***different***
    - It is not simply assuming something about leakage reactance



CIM5 will never match MOTORW for Double-Cage Motor

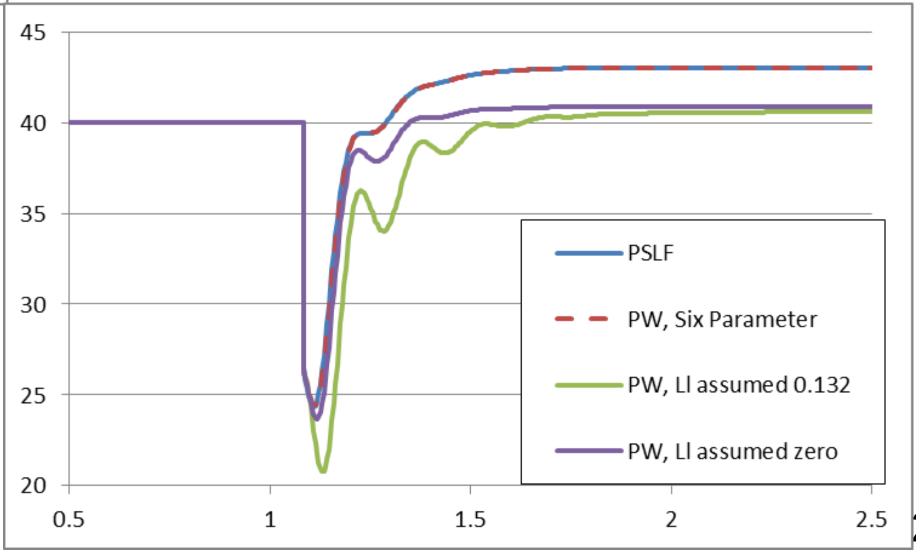


Work in 2014 by Dr. Tom Overbye at University of Illinois

No assumed value of Ll gives a match
It's a different model!

2014: PowerWorld added an option implementing MOTORW using the PSLF block diagram

Different Model – now it matches



Side Track: MOTORW Double Cage Model

- Present Software Status
  - MOTORW is a PSLF model. The CMPLDW model in PSLF uses this as component for 3-phase motors
  - PowerWorld Simulator and PowerTech TSAT have implemented the ability to use these MOTORW equations only to match the results our customers who use PSLF see
  - Siemens PSS/E has not added implementation these special MOTORW equations
- Summary
  - Move away from MOTORW
  - PSLF has a modular load component model named _cmp_mot3 which is based on MOTORW
  - PSLF needs to make a new one that is based on the correct 3-phase motor model instead and users can transition to that (maybe name it _cmp_cim5? and use circuit parameter input parameters?)

Summary

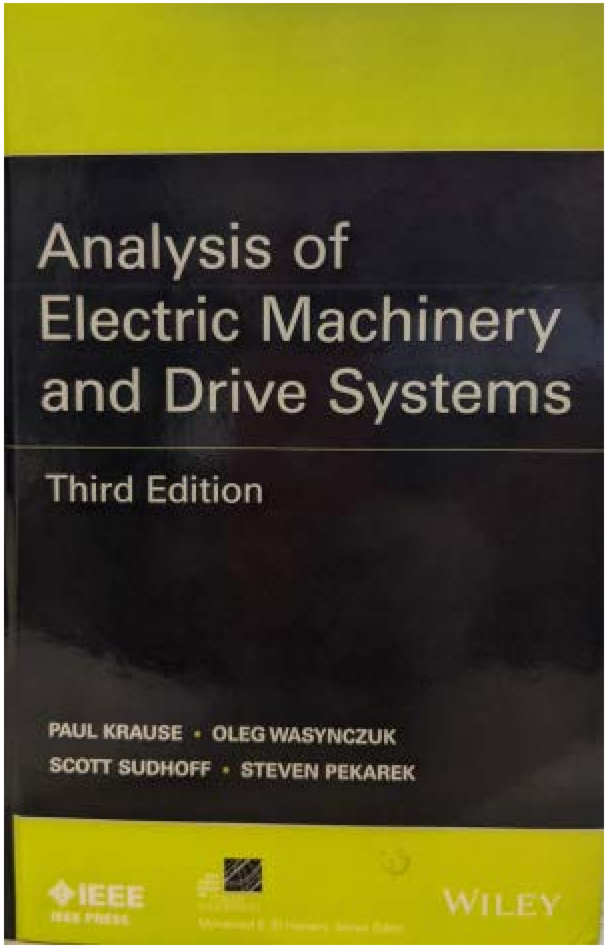
- 3-phase induction motors are 3-phase machines
  - Very similar to 3-phase synchronous machines
- Need to add new models that include frequency-dependence
- Transition away from using MOTORW

Following Slides: Frequency Dependence

- Following slides cover more theory on frequency dependence terms of induction motor
- Explains why they should be modeled

Proving this with Math: Krause Book

- Another book *Analysis of Electric Machine and Drive Systems*, Paul Krause, Oleg Wasynczuk, Scott Sudhoff, Steven Pekarek
  - First Edition published in 2003
  - Third Edition is 2013



Proof that Stator Electric Frequency should be used

- Start with Krause et. All equation 6.5-22 and 6.5-23 on page 227 of the book
- These equations are on an *arbitrary* reference frame (ω has not been chosen)

$$v_{qs} = r_{\boldsymbol{s}} i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt} \qquad 6.5\text{-}22$$

$$v_{ds} = r_{\boldsymbol{s}} i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt} \qquad 6.5\text{-}23$$

Synchronous Machine → ignoring stator transients

- For a synchronous machine we used the rotor reference frame and thus the choice of $(\omega = \omega_r)$ is made

$$v_{qs} = r_s i_{qs} + \frac{\omega_r}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_r}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt}$$

- We end up writing the following by assuming derivatives go quickly to zero in a new pseudo steady state

$$v_{qs} = r_s i_{qs} + \frac{\omega_r}{\omega_b}\psi_{ds}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_r}{\omega_b}\psi_{qs}$$

- This is a valid approximation for a *synchronous* machine in the *rotor* reference frame
  - This feels like it's always correct, but it depends on the reference frame and the type of machine

How about induction motor on Synchronous Reference Frame

- Induction Motor uses synchronous reference frame, so this choice is $\omega = \omega_b$

$$v_{qs} = r_s i_{qs} + \frac{\omega_b}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_b}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt}$$

- The induction motor models in all our software tools then did same approximation to ignore stator transients

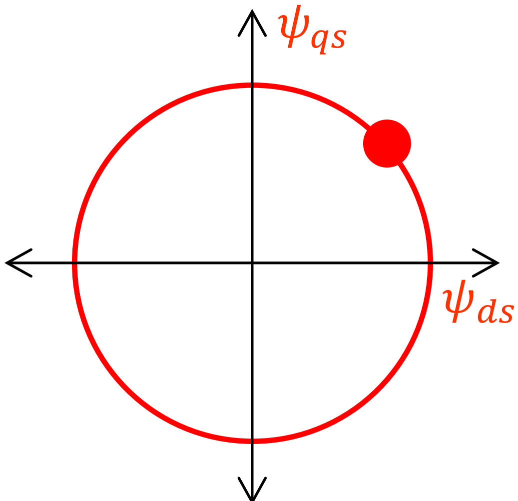
$$v_{qs} = r_s i_{qs} + \frac{\omega_b}{\omega_b}\psi_{ds}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_b}{\omega_b}\psi_{qs}$$

- Mistake was just made!
  - This is the source of our trouble
  - That's why Bernie's simulations are different
  - Can NOT just assume derivatives go to zero

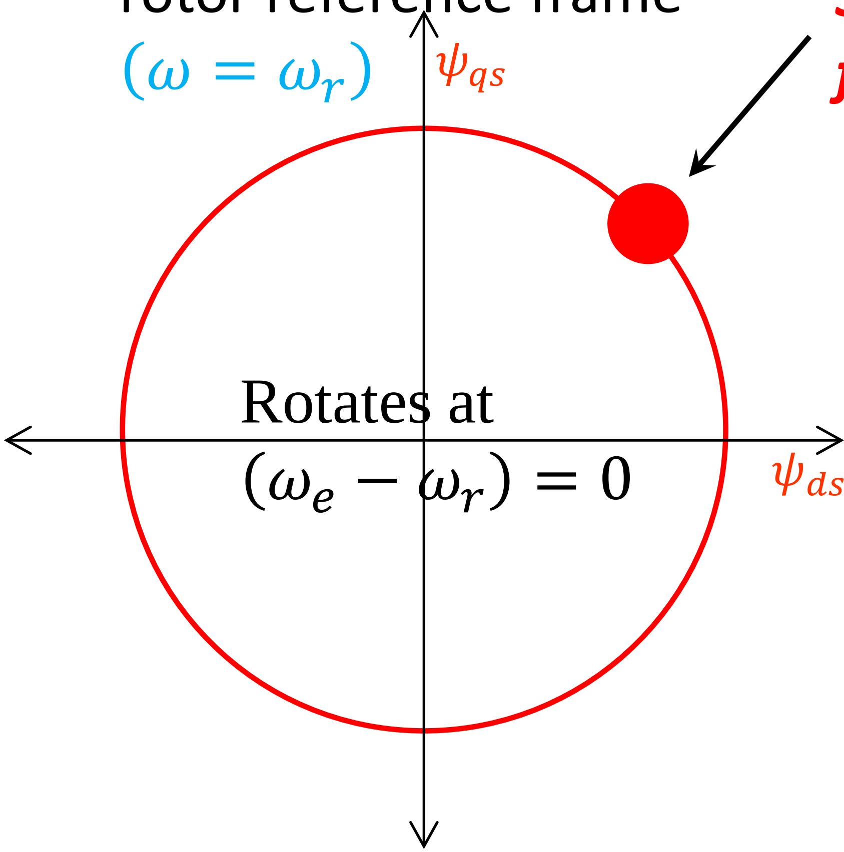
Do $d\psi_{qs}/dt$ and $d\psi_{qs}/dt$ always quickly go to zero? NO!

- Depends on the reference frame and the machine type
  - **Magnitude** reaches new steady state quickly with $\frac{d\psi_s}{dt} \rightarrow 0$
  - ψ_{qs} and ψ_{ds} reach a steady state spinning around a circle with a frequency of $\omega_e - \omega$
  - $\frac{d\psi_{qs}}{dt}$ and $\frac{d\psi_{ds}}{dt}$ derivatives become sinusoids and do NOT go to zero
- In rotor reference frame $\omega = \omega_r$, so that new steady state has the fluxes states spinning at $\omega_e - \omega_r$
  - In a synchronous machine, the new steady state will have $\omega_e = \omega_r$, thus that means the red dot becomes stationary
  - For synchronous machines it is correct that $\frac{d\psi_{qs}}{dt}$ and $\frac{d\psi_{ds}}{dt}$ would reach a new steady state and would go toward 0 quickly
  - This is NOT true for all machines and references



Synchronous Machine Graphical Explanation

Synchronous Machine at new steady state in rotor reference frame $(\omega = \omega_r)$



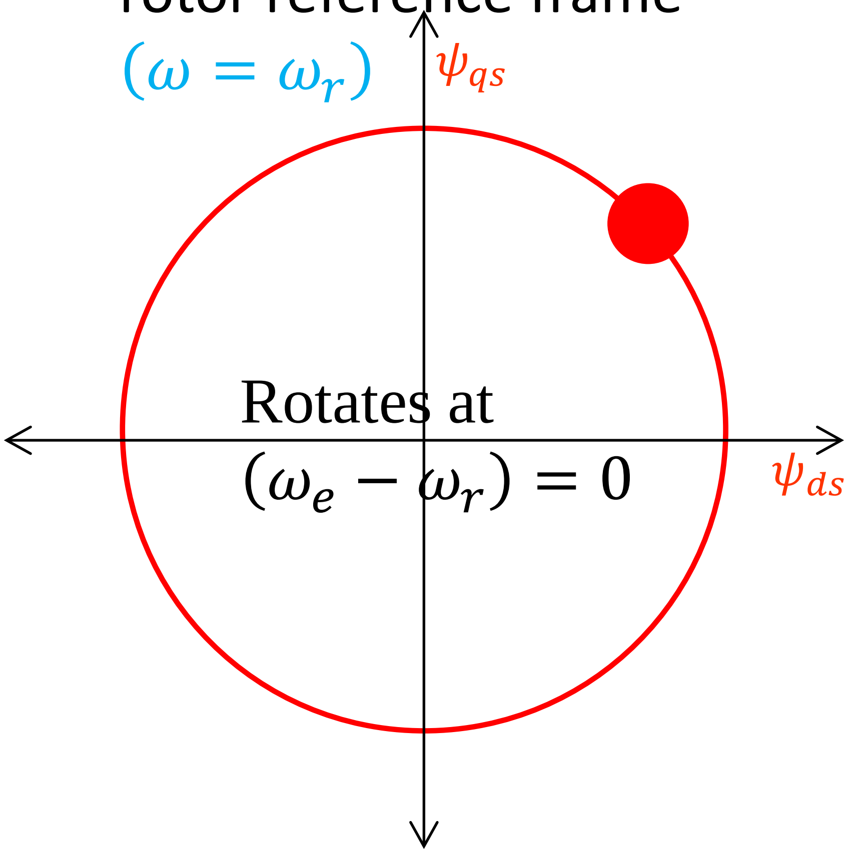
Steady State is reached with a fixed red dot that is stationary

Synchronous machine, so we assume $(\omega_r = \omega_e)$ is achieved very quickly

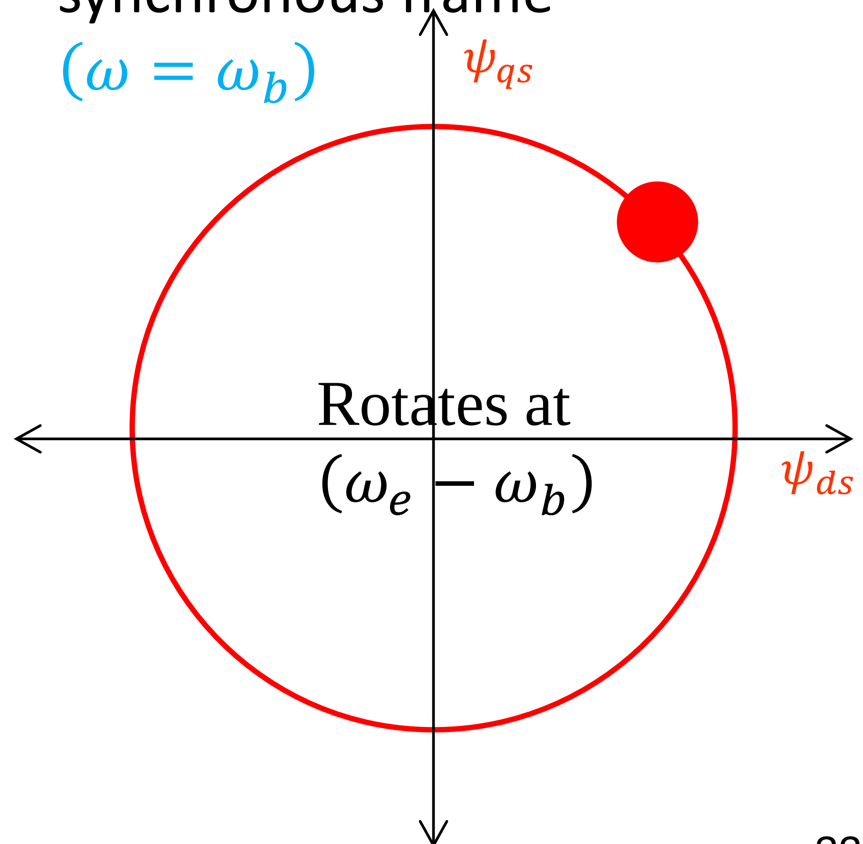
This is what "ignoring stator transients" means!

Compare to Induction Machine Graphical Explanation

Synchronous Machine at new steady state in rotor reference frame
$(\omega = \omega_r)$



Induction Machine at new steady state in synchronous frame
$(\omega = \omega_b)$



Algebra: Need to go back to the ABC phase phasors

ABC Phase Quantities

$$f_{as} = \sqrt{2} f_s \, cos(\theta_e)$$

$$f_{bs} = \sqrt{2} f_s \, cos\left(\theta_e - \frac{2\pi}{3}\right)$$

$$f_{cs} = \sqrt{2} f_s \, cos\left(\theta_e + \frac{2\pi}{3}\right)$$

$$\omega_e = \frac{d\theta_e}{dt}$$

After DQ transformation in an Arbitrary Reference Frame

$$f_{qs} = +\sqrt{2} f_s \cos(\theta_e - \theta)$$

$$f_{ds} = -\sqrt{2} f_s \sin(\theta_e - \theta)$$

$$f_{os} = 0$$

$$\omega = \frac{d\theta}{dt}$$

$$f_s = \frac{\sqrt{f_{qs}^2 + f_{ds}^2}}{\sqrt{2}}$$

Now Take Derivatives

$$f_{qs} = +\sqrt{2} f_s \cos(\theta_e - \theta)$$

$$f_{ds} = -\sqrt{2} f_s \sin(\theta_e - \theta)$$

$$\frac{df_{qs}}{dt} = \frac{d}{dt}\left[\sqrt{2} f_s\, cos(\theta_e - \theta)\right]$$
$$\frac{df_{qs}}{dt} = \frac{df_s}{dt}\left[\sqrt{2}\, cos(\theta_e - \theta)\right] - \sqrt{2} f_s\, sin(\theta_e - \theta)\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{qs}}{dt} = \frac{df_s}{dt}\left[\frac{\sqrt{2} f_s\, cos(\theta_e - \theta)}{f_s}\right] + \left[-\sqrt{2} f_s\, sin(\theta_e - \theta)\right]\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{qs}}{dt} = \frac{df_s}{dt}\frac{f_{qs}}{f_s} + f_{ds}(\omega_e - \omega)$$

$$\frac{df_{ds}}{dt} = \frac{d}{dt}\left[-\sqrt{2} f_s\, sin(\theta_e - \theta)\right]$$
$$\frac{df_{ds}}{dt} = \frac{df_s}{dt}\left[-\sqrt{2} f_s\, sin(\theta_e - \theta)\right] - \sqrt{2} f_s\, cos(\theta_e - \theta)\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{ds}}{dt} = \frac{df_s}{dt}\left[\frac{-\sqrt{2} f_s\, sin(\theta_e - \theta)}{f_s}\right] + \left[-\sqrt{2} f_s\, cos(\theta_e - \theta)\right]\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{ds}}{dt} = \frac{df_s}{dt}\frac{f_{ds}}{f_s} - f_{qs}(\omega_e - \omega)$$

Apply this to our equations in the *Arbitrary* Reference Frame

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt}$$

$$\boxed{\begin{aligned}\frac{d\psi_{qs}}{dt} &= \frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt} + \psi_{ds}(\omega_e - \omega)\\ \frac{d\psi_{ds}}{dt} &= \frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt} - \psi_{qs}(\omega_e - \omega)\end{aligned}}$$

Substitute in our derivative calculation

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt} + \psi_{ds}(\omega_e - \omega)\right]$$

$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt} - \psi_{qs}(\omega_e - \omega)\right]$$

Expand Terms

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{\omega_e}{\omega_b}\psi_{ds} - \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} - \frac{\omega_e}{\omega_b}\psi_{qs} + \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

Cancel Terms

$$v_{qs} = r_s i_{qs} + \frac{\omega_e}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_e}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

Ignore Stator Transients in any Reference Frame or Machine

- Approximation to ignore stator transients is

$$v_{qs} = r_s i_{qs} + \frac{\omega_e}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt}\right]$$
$$v_{ds} = r_s i_{ds} - \frac{\omega_e}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

Terms quickly go to zero regardless of reference frame choice and machine type

$$\frac{d\psi_s}{dt} \to 0$$

ψ_s is magnitude of flux phasor

- Result for an *arbitrary* reference frame

$$v_{qs} = r_s i_{qs} + \frac{\omega_e}{\omega_b}\psi_{ds}$$
$$v_{ds} = r_s i_{ds} - \frac{\omega_e}{\omega_b}\psi_{qs}$$

  - Use this for our induction machine!
  - Could also use this for a synchronous machine
    - Also valid to use $\frac{\omega_r}{\omega_b}$ for a synchronous machine though, so we do that because it's easier (ω_r is a state)