Steady-State Power System Security Analysis with PowerWorld Simulator

S1: Power System Modeling
Methods and Equations
Topics in the Section

• Nominal Voltage Levels
• Per Unit Values
• Admittance and Impedance
• Y-Bus Matrix
• Buses
• Transmission Branches
• Loads
• Switched Shunts
• Generators

• Power Flow Equations
• PV, PQ, Slack buses
• Newton's Method
• Multiple Solutions
• 2-Bus Power Flow
• PV and QV Curves
• Maximum Loadability
Transmission System: Nominal Voltage Levels

- Why do transmission systems operate at many different voltage levels?
  - Power = Voltage * Current
  - Thus for a given power, if you use a higher voltage, then the current will be lower
  - Why do we care? Transmission Losses
    - Losses = Resistance * (Current)^2
  - Example: What if we want to transmit 460 MW?
    - At 230 kV, that means 2,000 Amps
    - At 115 kV, that means 4,000 Amps
    - Because current is Twice as high for 115 kV, that means the losses would be 4 times higher

- Thus Higher Voltage is better, but more expensive, thus there is a trade-off
  - Means voltages vary depending on the situation
Transmission System: Nominal Voltage Levels

- Varying Nominal Voltage
  - Harder for a human to compare the voltage levels
  - Harder to handle in the equations used in power systems
    - You’d have to include “turns-ratio” multipliers all over the place

- This leads the industry to use a normalization method
Transmission Voltage Normalization using Per-Unit Values

- Per unit values are used in power systems to avoid worrying about various voltage level transitions created by transformers.
- They also allow us to compare voltages using a “percentage-like” number.
- All buses in the power system are assigned a Nominal Voltage.
  - Normally this is corresponds the physical voltage rating of devices connected to this bus and voltages are expected to be close to this.
    - This means 1.00 per unit voltage is usually “normal”.
  - But strictly speaking it doesn’t have to be. It’s just a number used to normalize the various parameters in the power system model.
    - Example: In Western United States, there is a lot of 500 kV transmission, but it generally operates at about 525 kV.
“Base Values”

- Define a “Power Base” (SBase) for the entire system
  - Transmission system SBase = 100 MVA
- The “Voltage Base” (VBase) for each part of the system is equal to the nominal voltage
- From these determine the “Impedance Base” (ZBase) and “Current Base” (IBase)

\[
\begin{align*}
Z_{\text{Base}} &= \frac{(V_{\text{Base}})^2}{S_{\text{Base}}} \\
I_{\text{Base}} &= \frac{S_{\text{Base}}}{\sqrt{3} V_{\text{Base}}}
\end{align*}
\]
Current Base Calculation: Line to Line Voltage

• Nominal Voltages are specified as the “Line to Line” voltage by tradition
  – This is the voltage magnitude difference between the A-B phase, B-C phase, and C-A phase
    \[ V_{\text{line-to-ground}} = \frac{V_{\text{line-to-line}}}{\sqrt{3}} \]
  • 138 kV \( \rightarrow \) 80 kV Line-Ground

• Current applies to only one phase, but the Power is the sum across all three phases, thus

\[
S = 3IV_{\text{line-to-ground}} \\
S = \sqrt{3}IV_{\text{line-to-line}} \\
I = \frac{S}{\sqrt{3}V_{\text{line-to-line}}} \\
I_{\text{Rating}} = \frac{MVARating}{\sqrt{3}\text{NomVoltage}}
\]
Voltage Base Zones

NOTE: Zones are separated by transformers
Determining a Per-Unit Value

- To determine a per-unit value, simply divide the actual number by the base.
- For example

\[ Z = 10 + j50 \text{ ohms} \]

\[ Z_{\text{Base}} = \frac{(138,000)^2}{100,000,000} = 190.44 \text{ Ohms} \]

\[ Z_{\text{pu}} = \frac{Z}{Z_{\text{Base}}} = \frac{10 + j50 \text{ Ohms}}{190.44 \text{ Ohms}} = 0.0525 + j0.2625 \]
The Transmission System - Model of the Wires

- Y-Bus (Admittance Matrix)
- Will review the various parts of the transmission system
- How we model transmission system
- How these models are entered into software
Impedance (Z) and Admittance (Y) and related terms R, X, B, and G

• The complex number for *Impedance* is represented by the letter Z
  
  \[ Z = R + jX \]
  
  • \( R = \text{Resistance} \)
  
  • \( X = \text{Reactance} \)

• *Admittance* is the numeric inverse of *Impedance* and represented by the letter Y
  
  \[ Y = G + jB \]
  
  • \( G = \text{Conductance} \)
  
  • \( B = \text{Susceptance} \)
Conversion Between Impedance (Z) and Admittance (Y)

- Impedance and Admittance are complex numbers and are inverses of each other

\[ y = \frac{1}{z} = \frac{1}{r + jx} = \left( \frac{r}{r^2 + x^2} \right) + j \left( \frac{-x}{r^2 + x^2} \right) \]

\[ g = \left( \frac{r}{r^2 + x^2} \right) \quad b = \left( \frac{-x}{r^2 + x^2} \right) \]
Y-Bus Matrix
(the Admittance Matrix)

- Used to model ALL of the transmission lines, transformers, capacitors, etc.
- These are all the “passive” elements
  - This part of the model does not change for different solution states
  - Represents only constant impedances
- The Y-Bus is an N X N matrix
  - N is the number of buses in the system
- A software package will calculate the Y-Bus from the data provided by the user regarding the passive elements of the system
  - Transmission Lines
  - Line Shunts
  - Switched Shunts (Capacitors/Reactors)
  - Bus Shunts
Power System Bus (or node)

- Nominal Voltage (in kV)
- B Shunt, G Shunt (in Nominal Mvar)
- Voltage Magnitude in per unit (calculated)
  - Used as initial guess in power flow solution
- Voltage Angle in degrees (calculated)
  - Used as initial guess in power flow solution
Convert "Nominal MW/Mvar" into a per unit admittance

- G Shunt and B Shunt are given as MW or Mvar at Nominal Voltage $(G_{\text{Shunt}}_{\text{MW}}$ and $B_{\text{Shunt}}_{\text{MVar}})$
- They represent constant admittance $G + jB$
- Writing this in actual units
- Converting to Per Unit Values

$$B_{\text{Shunt}}_{\text{MVar}} = V_{\text{nom}}^2 B$$

$$B_{pu} = +\left(\frac{B_{\text{Shunt}}_{\text{MVar}}}{S_{\text{Base}}}\right) \left(\frac{V_{\text{nom}}}{V_{\text{nom}}}\right)^2 = \frac{B_{\text{Shunt}}_{\text{MVar}}}{S_{\text{Base}}}$$

- Similar derivation for G
Bus Shunts (B and G)

- Values expressed in MW/Mvar at nominal voltage
- Represent a constant impedance/admittance at bus
  - $B$ Shunt: represents an impedance Mvar \textit{injection}
  - $G$ Shunt: represents an impedance MW \textit{absorption}
    - Sign of $B$ and $G$ are opposite for historical reasons
      - $G$ represented a load term
      - $B$ represented a capacitor term
- Y-Bus is affected only on diagonal

$\begin{bmatrix}
  \frac{G_{\text{Shunt}}_{\text{MW}}}{S_{\text{Base}}} & j\frac{B_{\text{Shunt}}_{\text{MVar}}}{S_{\text{Base}}} \\
  \vdots & \ddots & \vdots \\
  \frac{G_{\text{Shunt}}_{\text{MW}}}{S_{\text{Base}}} & j\frac{B_{\text{Shunt}}_{\text{MVar}}}{S_{\text{Base}}} & \ddots & \vdots \\
  \vdots & \ddots & \ddots & \ddots
\end{bmatrix}$

On Bus Dialog
Transmission Branch (Line or Transformer)

• Impedance Parameters
  – Series Resistance (R) in per unit
  – Series Reactance (X) in per unit
  – Shunt Charging (B) in per unit
  – Shunt Conductance (G) in per unit

Note: This model is modified when including tap-ratios or phase-shifts for variable transformers
Transmission Branch
affect on the Y-Bus

• To make the Y-Bus, we express all the impedances of the model as an admittance

\[
g_{\text{series}} = \left( \frac{r}{r^2 + x^2} \right) \quad b_{\text{series}} = \left( \frac{-x}{r^2 + x^2} \right)
\]

• Then add several terms to the Y-Bus as a result of the transmission line (or transformer)
Switched Shunts (Capacitor and Reactor Banks)

- Input the nominal MVAR, the MVAR supplied by the capacitor *at nominal voltage*
- It represents a constant impedance
- Use same conversion as with the Bus Shunts
- Y-Bus is thus affected only on diagonal terms

On Switched Shunt Dialog

![Nominal Mvar](108.0)
Actual Mvar

116.7
What does Y-Bus look like?

- Graphic represents a 118 Bus System
  - Sparse or “Mostly Zeros”
  - “Incident Symmetric”

Each dot represents a non-zero entry in the Y-Bus
Loads (ZIP model)

• Loads are modeled as constant impedance (Z), current (I), power (P), or a combination of the three.
• For each of these values you specify a real power and a reactive power \( (P_{\text{spec}} + jQ_{\text{spec}}) \).
• Constant Power - the most commonly used.

\[
S_{Lk} = P_{\text{spec}} + jQ_{\text{spec}}
\]
Constant Current Load

- P+jQ specified are what the load would be at nominal voltage of 1.0 per unit
- Go back to the equations for complex power and derive expression for S

\[ S = VI^* \quad \Rightarrow \quad I^* = \frac{S}{V} = \frac{P_{\text{spec}} + jQ_{\text{spec}}}{1.0 \angle \theta_v} \quad \Rightarrow \quad S = V \angle \theta_v \left( \frac{P_{\text{spec}} + jQ_{\text{spec}}}{1.0 \angle \theta_v} \right) \]

- Load becomes a linear function of voltage magnitude

\[ S_{Lk} = |V| \left( P_{\text{spec}} + jQ_{\text{spec}} \right) \]
**Constant Impedance Load**

- P+jQ specified are what the load would be at nominal voltage of 1.0 per unit
- Go back to the equations for complex power

\[ S = VI^* = V \left( \frac{V}{Z} \right)^* = \frac{V^2}{Z^*} \Rightarrow Z^* = \frac{V^2}{S} = \frac{(1.0)^2}{P_{\text{spec}} + jQ_{\text{spec}}} \]

\[ S_{\text{load}} = \frac{V^2}{\left( \frac{(1.0)^2}{P_{\text{spec}} + jQ_{\text{spec}}} \right)} = V^2 \left( P_{\text{spec}} + jQ_{\text{spec}} \right) \]

- Load becomes a quadratic function of voltage magnitude

\[ S_{\text{Lk}} = \left| V \right|^2 \left( P_{\text{spec}} + jQ_{\text{spec}} \right) \]
Generators

• Model them as a source of Real and Reactive Power - MW, MVAR output

• Control features of generators
  – AVR (Automatic Voltage Regulation)
    • Controls the reactive power output (Q) to maintain a specified voltage level at a regulated bus (doesn’t have to be the bus to which it is connected)
  – AGC (Automatic Generation Control)
    • Modifies the real power output (P)
Generator MW Power Control Input

- MW Output
- Minimum and Maximum MW Output
- Available for AGC
- Enforce Limits
- Participation Factor
  - Used when on participation factor control

On Generator Dialog

<table>
<thead>
<tr>
<th>Power Control</th>
<th>MW Output</th>
<th>Min. MW Output</th>
<th>Max. MW Output</th>
<th>Available for AGC</th>
<th>Enforce MW Limits</th>
<th>Part. Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>162.100</td>
<td>0.000</td>
<td>162.500</td>
<td></td>
<td></td>
<td>2.34</td>
</tr>
</tbody>
</table>
Generator Voltage Control Features (AVR)

- Mvar Output
- Minimum and Maximum MVAR output
  - Or can Use Capability Curve
  - More detailed, but more data
- Voltage Setpoint in per unit
- Available for AVR
- Remote Regulation %
  - Used when more than one generator is controlling the same bus voltage

Must Stay inside shaded region during operation
The Power Flow Equations

• This is the heart of all power system analysis
• Kirchoff’s Laws for a power system:
  Sum of the Currents at every Node Equals Zero
Why the “Power Flow”?

• Many loads (like heaters) are just big resistors which should be just an impedance, so why solve the “power flow”?
  – Because our input data is MW and MVAR values. This is because ...
  – The experience of utilities for more than 100 years shows that consumers behave similar to constant power over the long-run
  – If the voltage drops on a heater, the power consumption also decreases, but eventually some people get cold and turn up the heat
    • Actually the built in thermostat and control system probably do this automatically
Deriving the Power Flow Equations

• Using the Y-Bus, write Kirchoff’s Current Law at every bus as a matrix equation

\[ YV - I_{\text{generators}} + I_{\text{loads}} = 0 \]

• We are studying POWER systems, so we like to talk about power not current, so

\[ S = VI^* = [V](V^*Y^*) - [V]I^{*\text{generators}} + [V]I^{*\text{loads}} = 0 \]

\[ [V](V^*Y^*) - S_{\text{generators}} + S_{\text{loads}} = 0 \quad \text{etc.} \]

• These are N-1 complex number equations (where \( N = \) the number of buses in the system)
The Power Flow Equations

- These N-1 complex number equations can then be written as 2*(N-1) real number equations as below:

\[
P_k = 0 = V_k \sum_{m=1}^{N-1} \left[ V_m [g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m)] \right] - P_{G_k} + P_{L_k}
\]

\[
Q_k = 0 = V_k \sum_{m=1}^{N-1} \left[ V_m [g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m)] \right] - Q_{G_k} + Q_{L_k}
\]

Note: The variables \( g_{km} \) and \( b_{km} \) are the real and imaginary parts of the Y-Bus.

\( Y = G + jB \quad \text{or} \quad Y_{km} = g_{km} + jb_{km} \)

- Transmission lines, transformers, capacitors
- Generators
- Loads
- Elements of the Y-Bus
The Power Flow Equation Variables

• Four parameters describe the bus
  – Voltage magnitude (V)
  – Voltage angle (δ or θ)
  – Real Power Injection \( (P = P_{\text{gen}} - P_{\text{load}}) \)
  – Reactive Power Injection \( (Q = Q_{\text{gen}} - Q_{\text{load}}) \)

• The objective of the Power Flow algorithm is to determine all four of these values
Three Types of Buses

- In the Power Flow we are given two of these values at each bus and then solve for the other two.
- Generally, there are three types of buses in a Power Flow.

<table>
<thead>
<tr>
<th>Type of Bus</th>
<th>Voltage Mag. (V)</th>
<th>Voltage Angle (δ)</th>
<th>Power Injection (P = P_{gen} – P_{load})</th>
<th>Reactive Power Injection (Q = Q_{gen} – Q_{load})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slack Bus (Vδ-Bus) (only one of these)</td>
<td>GIVEN</td>
<td>GIVEN</td>
<td>SOLVE FOR</td>
<td>SOLVE FOR</td>
</tr>
<tr>
<td>PV-Bus (generator on AVR control)</td>
<td>GIVEN</td>
<td>SOLVE FOR</td>
<td>GIVEN</td>
<td>SOLVE FOR</td>
</tr>
<tr>
<td>PQ-Bus (load or generator not on AVR control)</td>
<td>SOLVE FOR</td>
<td>SOLVE FOR</td>
<td>GIVEN</td>
<td>GIVEN</td>
</tr>
</tbody>
</table>
How does the Power Flow work?

• The power flow equations are non-linear, which means they cannot be directly solved
  – Linear: $35 = 7x - 14$, so $x = 7$
  – Non-linear: $5x = \sin(x)$, so $x = ???$

• To solve non-linear equations, an iterative technique must be used

• To solve the Power Flow, Newton’s Method is normally the best technique
  – Simulator uses this by default.
Solving Non-Linear Equations

• Power Flow Equations are NON-LINEAR equations.
  – There are cos(*) and sin(*) terms which make the equations non-linear
• This means that finding a direct solution to them is impossible
• Thus we must use iterative numerical schemes to determine their solution
• For power flow equations, variations on Newton’s Method has been found to be the best technique
Newton’s Method

• Discussed in Calculus courses
• Consider a simple scalar equation \( f(x) \)
• You can write \( f(x) \) as a “Taylor Series”

\[
f(x) = f(x_0) + \left( \frac{\partial f}{\partial x} \right)_{x=x_0} (x - x_0) + \frac{1}{2} \left( \frac{\partial^2 f}{\partial x^2} \right)_{x=x_0} (x - x_0)^2 + \text{h.o.t.}
\]
Now, approximate this Taylor Series by ignoring all but the first two terms:

\[ f(x) \approx f(x_0) + \left( \frac{\partial f}{\partial x} \right)_{x=x_0} (x - x_0) \]

\[ [x - x_0] \approx \left( \frac{\partial f}{\partial x} \right)_{x=x_0}^{-1} [f(x) - f(x_0)] \]

We are trying to find where \( f(x) = 0 \) (the power flow equations sum to zero), therefore we can approximate and estimate:

\[ x \approx x_0 - \left( \frac{\partial f}{\partial x} \right)_{x=x_0}^{-1} [f(x_0)] \]
Newton’s Method

• This is called an iterative method because you take a guess at \(x_0\) and use the “Newton-step” to a new guess called \(x_1\). Then you use \(x_1\) to find \(x_2\), etc.

• Consider \(f(x) = x^2 - 2\). We find

\[
\left( \frac{\partial f}{\partial x} \right)_{x=x_k} = 2x_k
\]

• Thus \(\Delta x_k \approx -\frac{1}{2x_k} (x_k^2 - 2)\) and

\[
x_{k+1} \approx x_k + \Delta x_k = x_k + \left( -\frac{1}{2x_k} (x_k^2 - 2) \right)
\]
Newton’s Method

• Now take an initial guess, say $x_0 = 1$, and iterate the previous equation

<table>
<thead>
<tr>
<th>$k$</th>
<th>$x_k$</th>
<th>$f(x_k)$</th>
<th>$\Delta x_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>-1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>0.25</td>
<td>-0.08333</td>
</tr>
<tr>
<td>2</td>
<td>1.41667</td>
<td>6.953x10^{-3}</td>
<td>-2.454x10^{-3}</td>
</tr>
<tr>
<td>3</td>
<td>1.41422</td>
<td>6.024x10^{-6}</td>
<td>Done</td>
</tr>
</tbody>
</table>

• Error decreases quite quickly
  – Quadratic convergence
• $f(x_k)$ is known as the “mismatch”
  – The problem is solved when the mismatch is zero
Non-Linear Equations can have Multiple Solutions

- Consider the previous example.
  - There are two solutions. \( x = \sqrt{2} \) and \( x = -\sqrt{2} \)

- In an iterative scheme, the initial guess determines which solutions you “converge” to. Consider starting at \( x_0 = -1 \).

<table>
<thead>
<tr>
<th>k</th>
<th>( x_k )</th>
<th>( f(x_k) )</th>
<th>( \Delta x_k )</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<td>6.024x10^{-6}</td>
<td>Done</td>
</tr>
</tbody>
</table>

Converged to \( x = -\sqrt{2} \)
Non-Linear Equations can have No Real Solution

• Consider the equation $x^2 - \lambda = 0$, where $\lambda$ is an independent parameter
• The number of real solutions depends on the value of $\lambda$.
  – For $\lambda > 0$, there are two real solutions
  – For $\lambda = 0$, there is one real solution at $x=0$
  – For $\lambda < 0$, there are NO real solutions
• Parameter variation can change the number of solutions of nonlinear systems
Global convergence characteristics of the N-R algorithm are not easy to characterize.

The N-R algorithm converges quite quickly provided the initial guess is “close enough” to the solution.

However the algorithm doesn’t always converge to the “closest” solution.

Some initial guesses are just plain bad. For example, if $\frac{\partial f}{\partial x}$ is near zero (“ill-conditioned”), the process may diverge.
Extending Scalar to Vectors and Matrices

- Newton’s method may also be used when you are trying to find a solution where many functions are equal to zero.
- Let $\mathbf{f}(\mathbf{x})$ be a $n$-dimensional function and let $\mathbf{x}$ be an $n$-dimensional vector.

$$
\mathbf{f}(\mathbf{x}) = \begin{bmatrix}
    f_1(x) \\
    \vdots \\
    f_n(x)
\end{bmatrix} \quad \mathbf{x} = \begin{bmatrix}
    x_1 \\
    \vdots \\
    x_n
\end{bmatrix}
$$
Newton’s Method for Multiple Equations

• Now, the Newton-step is still defined as

\[ x_{k+1} - x_k = \Delta x_k \approx -\left( \frac{\partial f}{\partial x} \right)_{x=x_k}^{-1} f(x_k) \]

• But \( \frac{\partial f}{\partial x} \) is now a matrix, called the Jacobian

\[
\frac{\partial f}{\partial x} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \cdots & \cdots & \frac{\partial f_n}{\partial x_n}
\end{bmatrix}
\]
Solving the Power Flow Equations

• Power Flow Equations are NON-LINEAR equations
  – The cos(*) and sin(*) terms make them non-linear

• This means we have to use an iterative technique to solve them

• Define $\mathbf{f}$ to be the real and reactive power balance equations at every bus

• Define $\mathbf{x}$ to be the vector of voltages and angles at every bus (except the slack bus)
Setting up the Power Flow Solution

• Recall

\[ P_k = V_k \sum_{m=1}^{N-1} [V_m (g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m))] - P_Gk + P_{Lk} \]

\[ Q_k = V_k \sum_{m=1}^{N-1} [V_m (g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m))] - Q_{Gk} + Q_{Lk} \]

• Thus,

\[ f(x) = \begin{bmatrix} P_1 \\ Q_1 \\ P_2 \\ Q_2 \\ \vdots \\ P_{n-1} \\ Q_{n-1} \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} \delta_1 \\ V_1 \\ \delta_2 \\ V_2 \\ \vdots \\ \delta_{n-1} \\ V_{n-1} \end{bmatrix} \]

\[ \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial V_1} & \frac{\partial P_1}{\partial \delta_2} & \frac{\partial P_1}{\partial V_2} & \cdots & \frac{\partial P_1}{\partial \delta_{n-1}} & \frac{\partial P_1}{\partial V_{n-1}} \\ \frac{\partial Q_1}{\partial \delta_1} & \frac{\partial Q_1}{\partial V_1} & \frac{\partial Q_1}{\partial \delta_2} & \frac{\partial Q_1}{\partial V_2} & \cdots & \frac{\partial Q_1}{\partial \delta_{n-1}} & \frac{\partial Q_1}{\partial V_{n-1}} \\ \frac{\partial P_2}{\partial \delta_1} & \frac{\partial P_2}{\partial V_1} & \cdots & \cdots & \frac{\partial P_2}{\partial \delta_{n-1}} & \frac{\partial P_2}{\partial V_{n-1}} \\ \frac{\partial Q_2}{\partial \delta_1} & \frac{\partial Q_2}{\partial V_1} & \cdots & \cdots & \frac{\partial Q_2}{\partial \delta_{n-1}} & \frac{\partial Q_2}{\partial V_{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_{n-1}}{\partial \delta_1} & \frac{\partial P_{n-1}}{\partial V_1} & \cdots & \cdots & \frac{\partial P_{n-1}}{\partial \delta_{n-1}} & \frac{\partial P_{n-1}}{\partial V_{n-1}} \\ \frac{\partial Q_{n-1}}{\partial \delta_1} & \frac{\partial Q_{n-1}}{\partial V_1} & \cdots & \cdots & \frac{\partial Q_{n-1}}{\partial \delta_{n-1}} & \frac{\partial Q_{n-1}}{\partial V_{n-1}} \end{bmatrix} \]

Note: There are \( n \) buses, with bus \( n \) being the slack bus.
Power Flow Solutions

• The power flow equations exhibit the same behavior as other non-linear equations

• In theory there are up to $2^{n-1}$ solutions to the power flow equations
  – Of these, ALL or NONE may be real number solutions

• In power system analysis, we are normally interested only in the solution that is the “high voltage” solution for every bus
Consider this lossless (R=0) example system:

\[ R + jX = 0.000 + j0.100 \]

Variable Definitions:

- \( \theta \) = Voltage Angle at Bus 2
- \( V \) = Voltage Magnitude at Bus 2
- \( B = \left( \frac{-x}{r^2 + x^2} \right) = \left( \frac{-0.1}{0 + 0.1^2} \right) = -10 \)

Power balance equations:

- \( PMismatch = P_L - BV \sin(\theta) \)
- \( QMismatch = Q_L + BV \cos(\theta) - BV^2 \)
How do power flow solutions vary as load is changed?

• Solution: Calculate a series of power flow solutions for various load levels

• First determine the following

\[
\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \mathbf{J}(\theta, V) = \begin{bmatrix}
- BV \cos(\theta) & - B \sin(\theta) \\
- BV \sin(\theta) & B \cos(\theta) - 2BV
\end{bmatrix}
\]

\[
\mathbf{x}_k = \begin{bmatrix}
\theta \\
V
\end{bmatrix} ; \quad \mathbf{f}(\mathbf{x}) = \begin{bmatrix}
P_L - BV \sin(\theta) \\
Q_L + BV \cos(\theta) - BV^2
\end{bmatrix}
\]
Example Power Flow Solution

• Now for each load level, iterate from an initial guess until the solution (hopefully) converges

\[ x \approx x_0 - \left( \frac{\partial f}{\partial x} \right)_{x=x_0}^{-1} [f(x_0)] \]

• Consider a per unit load of \( P_L = 2 \text{ pu} \) and \( Q_L = 0 \text{ pu} \)
  – A 200 MW load since \( S_{\text{Base}} = 100 \text{ MVA} \)
• Then consider a “flat start” for the initial guess: \( V_0 = 1 ; \theta_0 = 0 \).
• Also consider a start of \( V_0 = 0.5 ; \theta_0 = -1 \).
Simple 2-Bus Power Flow in Excel

- Open Microsoft Excel
  - \text{...\S01\_SystemModeling\Power Flow Two Bus.xls}
  - Gray Shaded cells are user inputs (initial guess, load and network parameters)

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
| 1 | Network Parameters | | | | | | | | | | | | | | | | | | |
| 2 | \( P_L \) (MW) | | | | | | | | | | | | | | | | | | |
| 3 | \( Q_L \) (MVar) | | | | | | | | | | | | | | | | | | |
| 4 | \( B \) | | | | | | | | | | | | | | | | | | |
| 5 | \( \theta \) | | | | | | | | | | | | | | | | | | |
| 6 | \( V \) | | | | | | | | | | | | | | | | | | |

Variable Definitions:

\[
B = \left( \frac{-x}{r^2 + x^2} \right) = \left( \frac{-0.1}{0 + 0.1^2} \right) = -10
\]

\( \theta \) = Voltage Angle at Bus 2

\( V \) = Voltage Magnitude at Bus 2

Power Flow Mismatch Equations

\[
P_{\text{Mismatch}} = P_L - BV \sin(\theta)
\]

\[
Q_{\text{Mismatch}} = Q_L + BV \cos(\theta) - BV^2
\]

Using

\[
P_L = 2.0 \quad \Rightarrow \quad P_{\text{Mismatch}} = 2 + 10V \sin(\theta)
\]

\[
Q_L = 0 \quad \Rightarrow \quad Q_{\text{Mismatch}} = -10V \cos(\theta) + 10V^2
\]

\[
B = -10
\]

Power Flow Jacobian Matrix

\[
\frac{\mathbf{M}}{\partial \mathbf{x}} = J(\theta, V) = \begin{bmatrix}
- BV \cos(\theta) & -B \sin(\theta) \\
- B \sin(\theta) & - BV \cos(\theta) & -2BV
\end{bmatrix}
\]

Using

\[
P_L = 2.0 \quad \Rightarrow \quad \begin{bmatrix}
10V \cos(\theta) & 10 \sin(\theta) \\
10V \sin(\theta) & -10 \cos(\theta) + 20V
\end{bmatrix}
\]

\[
B = -10
\]
Iteration Results for Power Flow Solution

• Change Initial Voltage to $V_0 = 1 ; \theta_0 = 0$
  
  – Cell B5 = 1.0
  – Cell C5 = 0.0

<table>
<thead>
<tr>
<th>$k$</th>
<th>$V^k$</th>
<th>$\theta^k$ (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
<td>-0.0000</td>
</tr>
<tr>
<td>1</td>
<td>1.0000</td>
<td>-0.2000</td>
</tr>
<tr>
<td>2</td>
<td>0.9794</td>
<td>-0.2055</td>
</tr>
<tr>
<td>3</td>
<td>0.9789</td>
<td>-0.2058</td>
</tr>
<tr>
<td>4</td>
<td>0.9789</td>
<td>-0.2058</td>
</tr>
</tbody>
</table>

High Voltage Solution

• Change Initial Voltage to $V_0 = 0.5 ; \theta_0 = -1$

  – Cell B5 = 0.5
  – Cell C5 = -1.0

<table>
<thead>
<tr>
<th>$k$</th>
<th>$V^k$</th>
<th>$\theta^k$ (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5000</td>
<td>-1.0000</td>
</tr>
<tr>
<td>1</td>
<td>0.0723</td>
<td>-1.5152</td>
</tr>
<tr>
<td>2</td>
<td>0.2010</td>
<td>-1.3397</td>
</tr>
<tr>
<td>3</td>
<td>0.2042</td>
<td>-1.3657</td>
</tr>
<tr>
<td>4</td>
<td>0.2043</td>
<td>-1.3650</td>
</tr>
</tbody>
</table>

Low Voltage Solution
Region of Convergence

Initial guess of Bus 2 angle is plotted on x-axis, voltage magnitude on y-axis, for load = 200 MW, 100 Mvar; x = 0.1 pu

Initial guesses in the red region converge to the high voltage solution, while those in the yellow region converge to the low voltage solution.
A PV-Curve is generated by plotting these two solutions as the real power load is varied with the reactive load and impedance constant.

- Experiment with parameters in cells O2-O4.
- Higher transmission impedance usually decreases the system capacity (maximum load).

Note: load values below 500 have two solutions, those above 500 have no solution.

For a critical value (i.e. the “nose point”) there is only one solution.
QV-Curves

- A QV-Curve is generated by plotting these two solutions as the reactive power load is varied with the real load and impedance constant.

Again: load values below 250 have two solutions, while those above 250 have no solution.

For a critical value (i.e. the “nose point”) there is only one solution.
Maximum Loadability

• The “nose points” on the PV and QV curves are critical values
• They correspond to points of maximum loadability from a static point of view
  – Note: this only considers the static equations. Other problems can occur long before the power system reaches this points: lines overheating or system dynamic problems
• The critical values depend on the relative allocation between real and reactive power
Mathematical Characteristics of the Critical Value

• At the critical value, the power flow Jacobian will be singular
  – A singular matrix is one that has a zero determinant (a generalization of scalar case when \( \frac{df}{dx} = 0 \))

• A singular matrix CANNOT be inverted
  – Thus the Newton power flow does not work well near this point because it requires us to invert the Jacobian matrix

\[
x_{k+1} - x_k = \Delta x_k \approx -\left( \frac{\partial f}{\partial x} \right)_{x=x_k}^{-1} f(x_k)
\]