Steady-State Power System Security Analysis with PowerWorld Simulator

S1: Power System Modeling

Methods and Equations

PowerWorld Corporation

2001 South First Street
Champaign, Illinois 61820
+1 (217) 384.6330

support@powerworld.com
http://www.powerworld.com

Topics in the Section

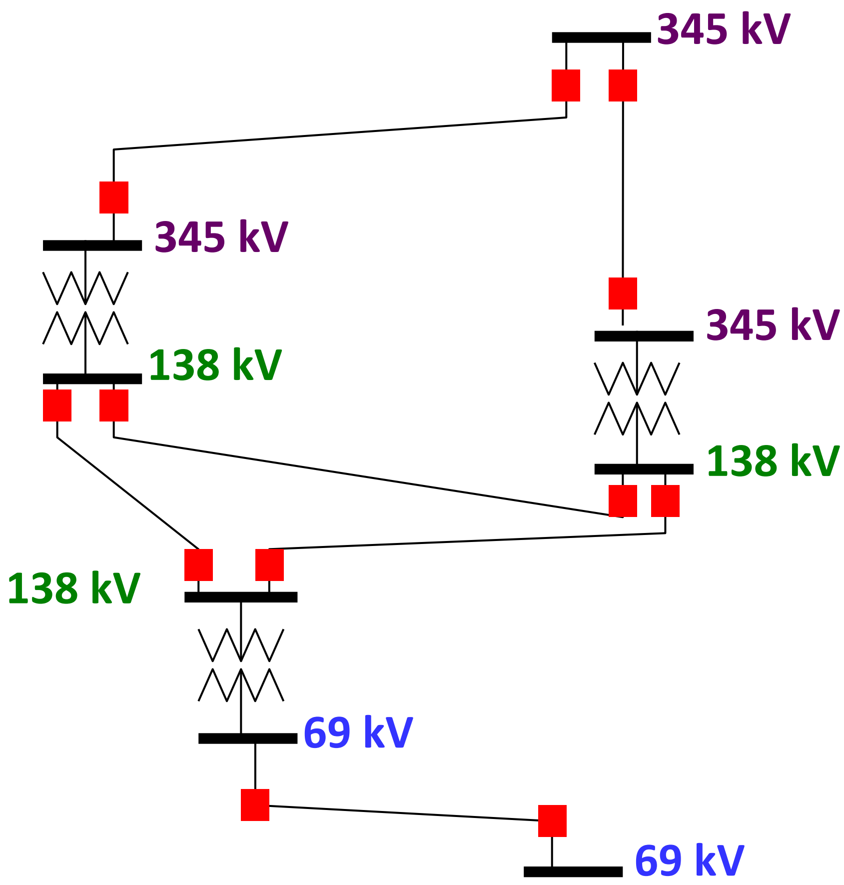
- Nominal Voltage Levels
- Per Unit Values
- Admittance and Impedance
- Y-Bus Matrix
- Buses
- Transmission Branches
- Loads
- Switched Shunts
- Generators
- Power Flow Equations
- PV, PQ, Slack buses
- Newton's Method
- Multiple Solutions
- 2-Bus Power Flow
- PV and QV Curves
- Maximum Loadability

Transmission System: Nominal Voltage Levels

- Why do transmission systems operate at many different voltage levels?
  - Power = Voltage * Current
    - Thus for a given power, if you use a higher voltage, then the current will be lower
  - Why do we care? Transmission Losses
    - Losses = Resistance * $(\text{Current})^2$
  - Example: What if we want to transmit 460 MW?
    - At 230 kV, that means 2,000 Amps
    - At 115 kV, that means 4,000 Amps
    - Because current is Twice as high for 115 kV, that means the losses would be 4 times higher
- Thus Higher Voltage is better, but more expensive, thus there is a trade-off
  - Means voltages vary depending on the situation

Transmission System: Nominal Voltage Levels

- Varying Nominal Voltage
  - Harder for a human to compare the voltage levels
  - Harder to handle in the equations used in power systems
    - You'd have to include "turns-ratio" multipliers all over the place
- This leads the industry to use a normalization method



Transmission Voltage Normalization using Per-Unit Values

- Per unit values are used in power systems to avoid worrying about various voltage level transitions created by transformers
- They also allow us to compare voltages using a "percentage-like" number
- All buses in the power system are assigned a Nominal Voltage.
  - Normally this is corresponds the physical voltage rating of devices connected to this bus and voltages are expected to be close to this
    - This means 1.00 per unit voltage is usually "normal"
  - But strictly speaking it doesn't have to be. It's just a number used to normalize the various parameters in the power system model.
    - Example: In Western United States, there is a lot of 500 kV transmission, but it generally operates at about 525 kV

“Base Values”

- Define a “Power Base” (SBase) for the entire system
  - Transmission system SBase = 100 MVA
- The “Voltage Base” (VBase) for each part of the system is equal to the nominal voltage
- From these determine the “Impedance Base” (ZBase) and “Current Base” (IBase)

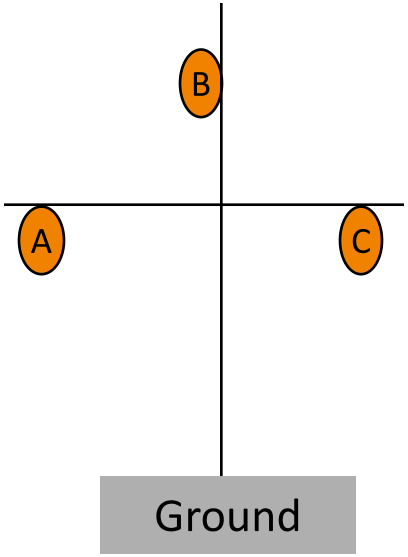
$$\text{ZBase} = \frac{(\text{VBase})^2}{\text{SBase}} \qquad \text{IBase} = \frac{\text{SBase}}{\sqrt{3}\ \text{VBase}}$$

Current Base Calculation: Line to Line Voltage

- Nominal Voltages are specified as the "Line to Line" voltage by tradition
  - This is the voltage magnitude difference between the A-B phase, B-C phase, and C-A phase

$$V_{line\text{-}to\text{-}ground} = \frac{V_{line-to-line}}{\sqrt{3}}$$

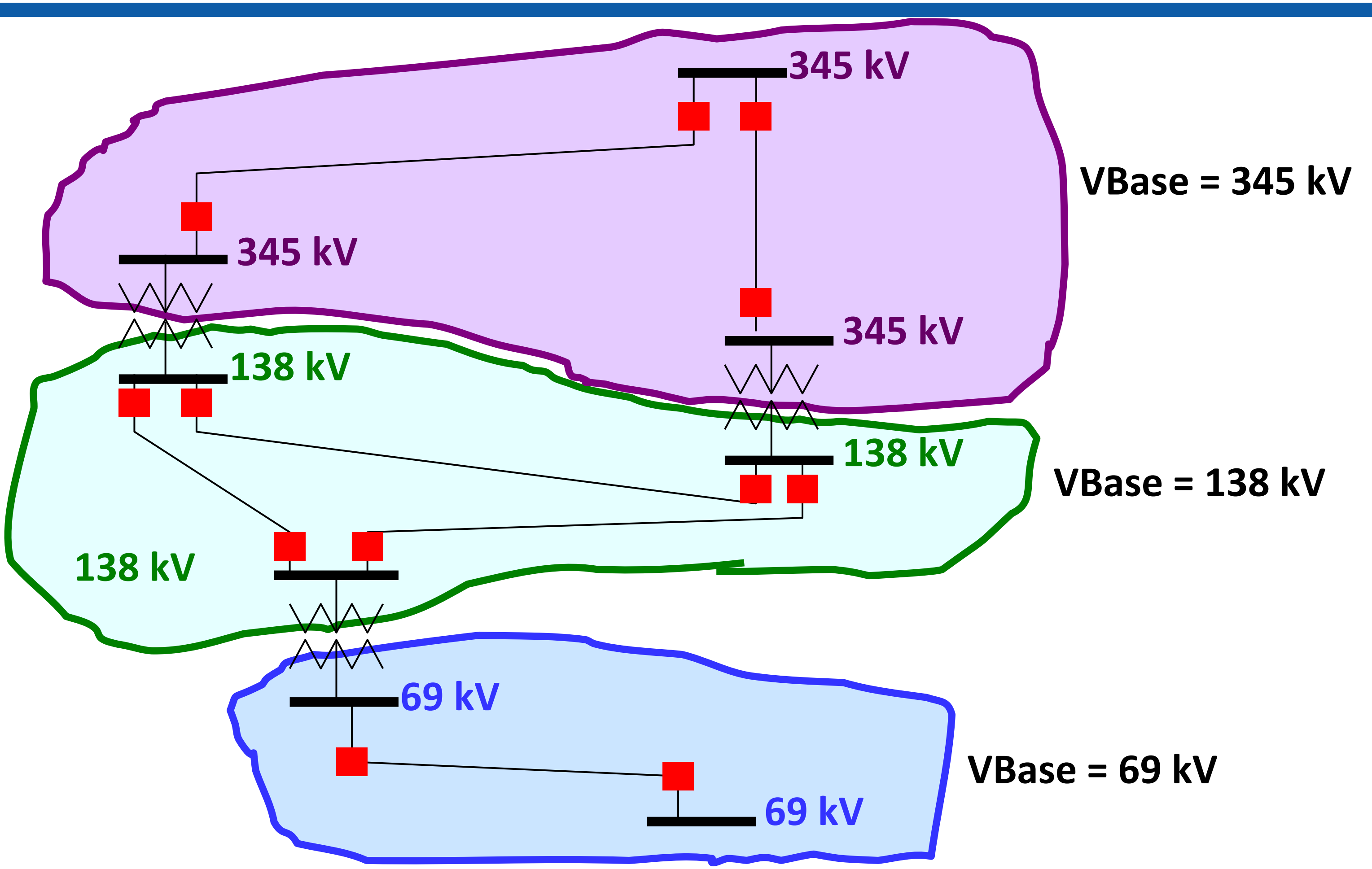
    - 138 kV → 80 kV Line-Ground



- Current applies to only one phase, but the Power is the sum across all three phases, thus

$$S = 3I\,V_{line\text{-}to\text{-}ground}$$
$$S = \sqrt{3}IV_{line-to-line}$$
$$\Rightarrow I = \frac{S}{\sqrt{3}V_{line-to-line}} \Rightarrow I_{Rating} = \frac{MVARating}{\sqrt{3}NomVoltage}$$

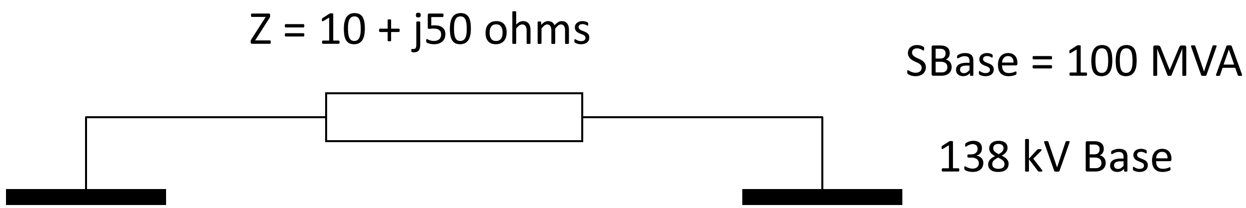
Voltage Base Zones



NOTE: Zones are separated by transformers

Determining a Per-Unit Value

- To determine a per-unit value, simply divide the actual number by the base
- For example



$$\text{ZBase} = \frac{(138{,}000)^2}{100{,}000{,}000} = 190.44\ \text{Ohms}$$

$$\text{Zpu} = \frac{\text{Z}}{\text{ZBase}} = \frac{10 + j50\ \text{Ohms}}{190.44\ \text{Ohms}} = 0.0525 + j0.2625$$

The Transmission System - Model of the Wires

- Y-Bus (Admittance Matrix)
- Will review the various parts of the transmission system
- How we model transmission system
- How these models are entered into software

Impedance (Z) and Admittance (Y) and related terms R, X, B, and G

- The complex number for *Impedance* is represented by the letter Z
  - Z = R + jX
    - R = *Resistance*
    - X = *Reactance*
- *Admittance* is the numeric inverse of *Impedance* and represented by the letter Y
  - Y = G + jB
    - G = *Conductance*
    - B = *Susceptance*

Conversion Between Impedance (Z) and Admittance (Y)

- Impedance and Admittance are complex numbers and are inverses of each other

$$y = \frac{1}{z} = \frac{1}{r + jx} = \underbrace{\left(\frac{r}{r^2 + x^2}\right)}_{g} + j\underbrace{\left(\frac{-x}{r^2 + x^2}\right)}_{b}$$

$$g = \left(\frac{r}{r^2 + x^2}\right) \qquad b = \left(\frac{-x}{r^2 + x^2}\right)$$

Y-Bus Matrix (the Admittance Matrix)

- Used to model ALL of the transmission lines, transformers, capacitors, etc.
- These are all the "passive" elements
  - This part of the model does not change for different solution states
  - Represents only constant impedances
- The Y-Bus is an N X N matrix
  - N is the number of buses in the system
- A software package will calculate the Y-Bus from the data provided by the user regarding the passive elements of the system
  - Transmission Lines
  - Line Shunts
  - Switched Shunts (Capacitors/Reactors)
  - Bus Shunts

Power System Bus (or node)

- Nominal Voltage (in kV)
- B Shunt, G Shunt (in Nominal Mvar)
- Voltage Magnitude in per unit (calculated)
  - Used as initial guess in power flow solution
- Voltage Angle in degrees (calculated)
  - Used as initial guess in power flow solution

Converting "Nominal MW/Mvar" into a per unit admittance

- G Shunt and B Shunt are given as MW or Mvar at Nominal Voltage $(GShunt_{MW} \quad and \quad BShunt_{MVar})$
- They represent constant admittance G + jB
- Writing this in actual units $BShunt_{MVar} = V_{nom}^2 B$
- Converting to Per Unit Values

$$B_{pu} = \frac{\dfrac{BShunt_{MVar}}{SBase}}{\left(\dfrac{V_{nom}}{V_{nom}}\right)^2} = \frac{BShunt_{MVar}}{SBase}$$

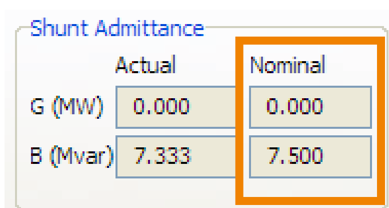
$$B_{pu} = +\left(\frac{BShunt_{MVar}}{SBase}\right)$$

- Similar derivation for G

Bus Shunts (B and G)

- Values expressed in MW/Mvar at nominal voltage
- Represent a constant impedance/admittance at bus
  - B Shunt : represents an impedance Mvar *injection*
  - G Shunt : represents an impedance MW *absorption*
    - Sign of B and G are opposite for historical reasons
      - G represented a load term
      - B represented a capacitor term
- Y-Bus is affected only on diagonal

On Bus Dialog



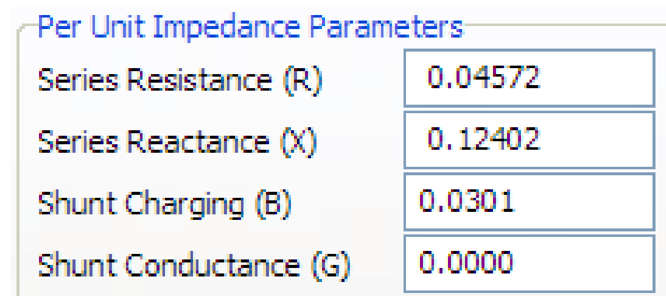
Bus K

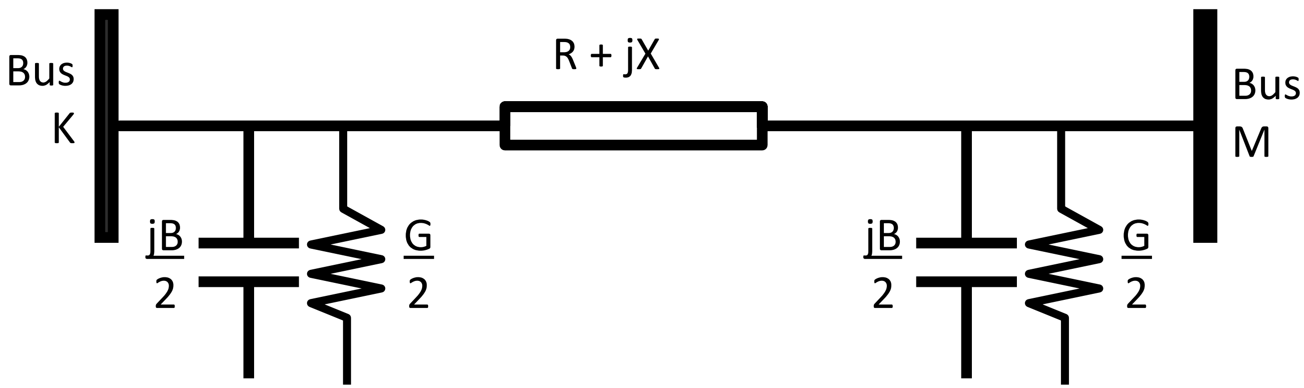
$$\text{Bus K}\begin{bmatrix} \ddots & \vdots & \cdots \\ \cdots & -\left(\frac{GShunt_{MW}}{SBase}\right)+j\left(\frac{BShunt_{MVar}}{SBase}\right) & \cdots \\ \vdots & \cdots & \ddots \end{bmatrix}$$

Transmission Branch (Line or Transformer)

- Impedance Parameters
  - Series Resistance (R) in per unit
  - Series Reactance (X) in per unit
  - Shunt Charging (B) in per unit
  - Shunt Conductance (G) in per unit

On Branch Dialog





Note: This model is modified when including tap-ratios or phase-shifts for variable transformers

Transmission Branch affect on the Y-Bus

- To make the Y-Bus, we express all the impedances of the model as an admittance

$$g_{series} = \left(\frac{r}{r^2 + x^2} \right) \qquad b_{series} = \left(\frac{-x}{r^2 + x^2} \right)$$

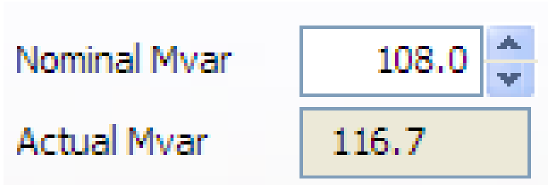
- Then add several terms to the Y-Bus as a result of the transmission line (or transformer)

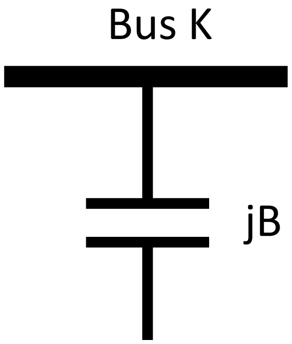
$$\begin{array}{c} \\ \vdots \\ \text{BusK} \\ \vdots \\ \text{BusM} \\ \vdots \end{array} \begin{array}{c} \cdots \quad \text{BusK} \quad \cdots \quad \text{BusM} \quad \cdots \\ \left[\begin{array}{c|c|c|c|c} \ddots & \cdots & \cdots & \cdots & \cdots \\ \hline \vdots & \left(g_{series} + jb_{series} + \frac{G}{2} + j\frac{B}{2} \right) & \cdots & \left(-g_{series} - jb_{series} \right) & \vdots \\ \hline \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline \vdots & \left(-g_{series} - jb_{series} \right) & \cdots & \left(g_{series} + jb_{series} + \frac{G}{2} + j\frac{B}{2} \right) & \vdots \\ \hline \vdots & \cdots & \cdots & \cdots & \ddots \end{array} \right] \end{array}$$

Switched Shunts (Capacitor and Reactor Banks)

- Input the nominal MVAR, the MVAR supplied by the capacitor *at nominal voltage*
- It represents a constant impedance
- Use same conversion as with the Bus Shunts
- Y-Bus is thus affected only on diagonal terms

On Switched Shunt Dialog



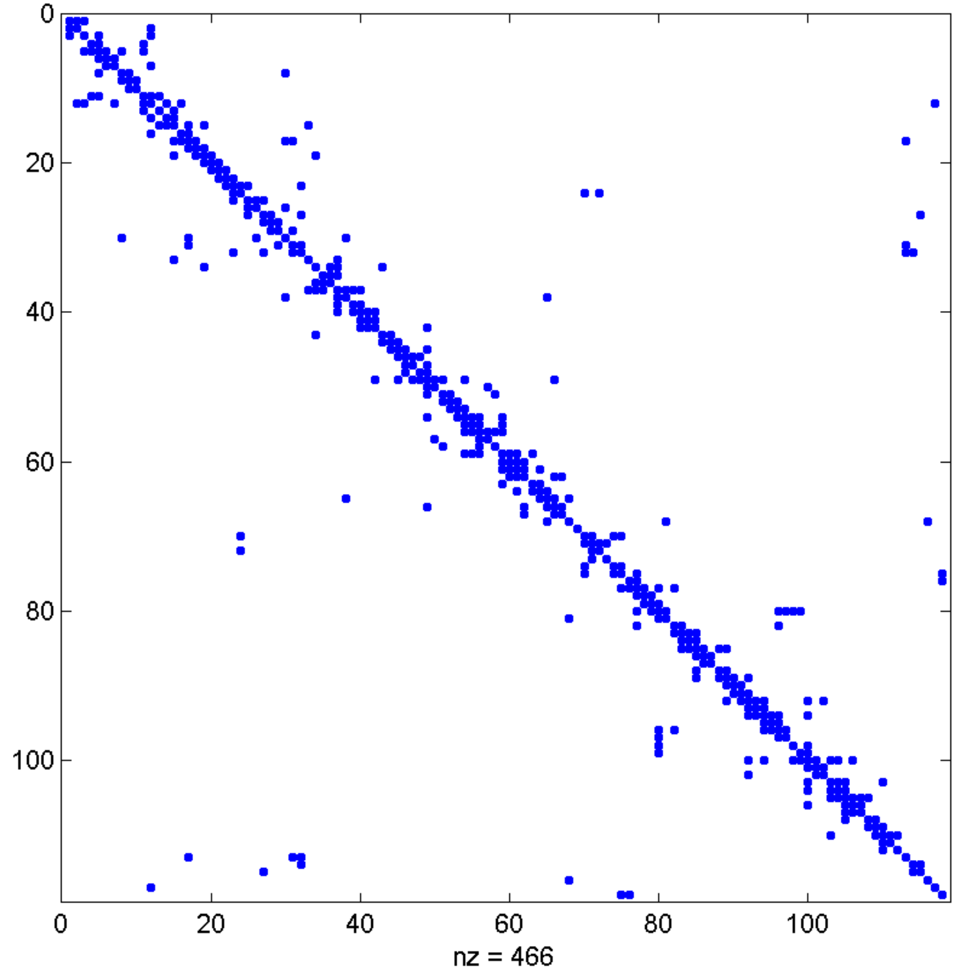


$$\begin{array}{c} \\ \text{Bus K} \\ \\ \end{array} \quad \begin{array}{c} \text{Bus K} \\ \left[\begin{array}{c|c|c} \ddots & \cdots & \cdots \\ \hline \vdots & +j\left(\dfrac{Q_{MVar}}{SBase}\right) & \vdots \\ \hline \vdots & \cdots & \ddots \end{array}\right] \end{array}$$

What does Y-Bus look like?

- Graphic represents a 118 Bus System
  - Sparse or “Mostly Zeros”
  - “Incident Symmetric”

Each dot represents a non-zero entry in the Y-Bus

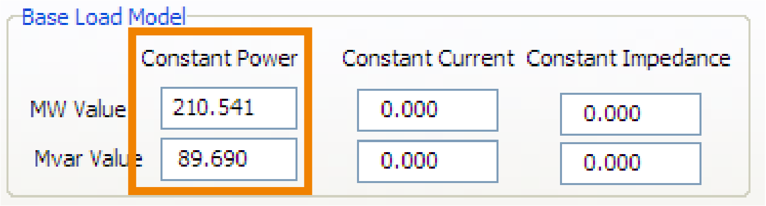


Loads (ZIP model)

- Loads are modeled as constant impedance (Z), current (I), power (P), or a combination of the three
- For each of these values you specify a real power and a reactive power ($P_{spec} + jQ_{spec}$)
- Constant Power - the most commonly used

On Load Dialog

$$\mathbf{S}_{Lk} = P_{spec} + jQ_{spec}$$



Constant Current Load

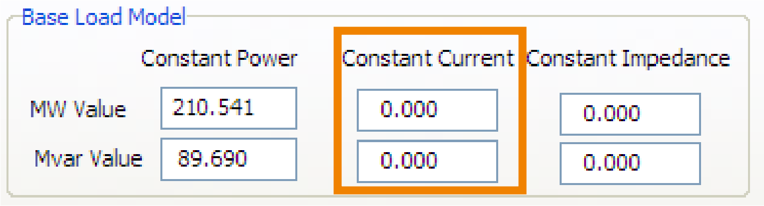
- P+jQ specified are what the load would be at nominal voltage of 1.0 per unit
- Go back to the equations for complex power and derive expression for S

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* \quad \Rightarrow \quad \mathbf{I}^* = \frac{\mathbf{S}}{\mathbf{V}} = \frac{\mathrm{P_{spec}} + \mathrm{jQ_{spec}}}{1.0\angle\theta_v} \quad \Rightarrow \quad \mathbf{S} = V\angle\theta_v\left(\frac{\mathrm{P_{spec}} + \mathrm{jQ_{spec}}}{1.0\angle\theta_v}\right)$$

- Load becomes a linear function of voltage magnitude

$$\mathbf{S}_{\mathrm{Lk}} = |V|(\mathrm{P_{spec}} + \mathrm{jQ_{spec}})$$

On Load Dialog



Constant Impedance Load

- P+jQ specified are what the load would be at nominal voltage of 1.0 per unit
- Go back to the equations for complex power

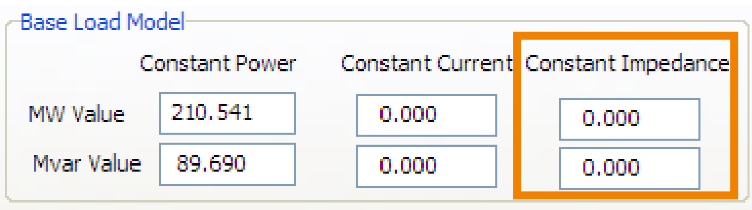
$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \mathbf{V}\left(\frac{\mathbf{V}}{\mathbf{Z}}\right)^* = \frac{V^2}{\mathbf{Z}^*} \Rightarrow \mathbf{Z}^* = \frac{V^2}{\mathbf{S}} = \frac{(1.0)^2}{\mathrm{P_{spec}} + j\mathrm{Q_{spec}}}$$

$$\mathbf{S}_{\mathrm{load}} = \frac{V^2}{\left(\frac{(1.0)^2}{\mathrm{P_{spec}} + j\mathrm{Q_{spec}}}\right)} = V^2(\mathrm{P_{spec}} + j\mathrm{Q_{spec}})$$

- Load becomes a quadratic function of voltage magnitude

$$\mathbf{S}_{\mathrm{Lk}} = |V|^2(\mathrm{P_{spec}} + j\mathrm{Q_{spec}})$$

On Load Dialog



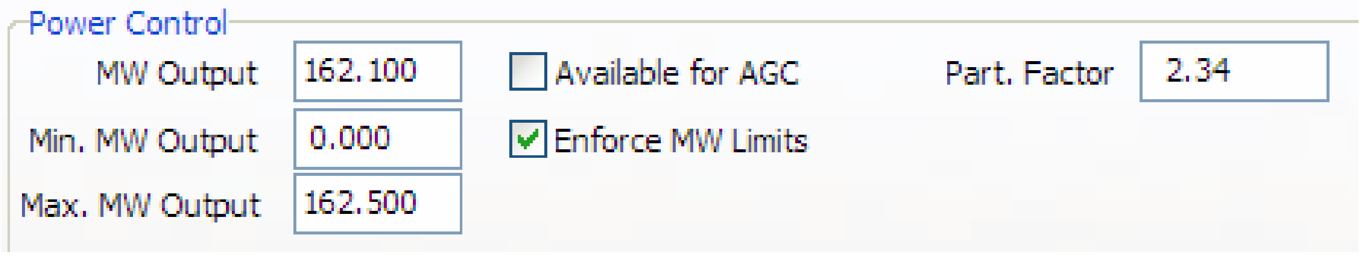
Generators

- Model them as a source of Real and Reactive Power - MW, MVAR output
- Control features of generators
  - AVR (Automatic Voltage Regulation)
    - Controls the reactive power output (Q) to maintain a specified voltage level at a regulated bus (doesn't have to be the bus to which it is connected)
  - AGC (Automatic Generation Control)
    - Modifies the real power output (P)

Generator MW Power Control Input

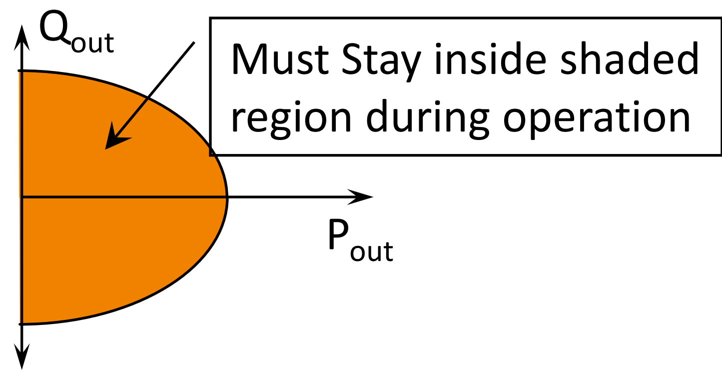
- MW Output
- Minimum and Maximum MW Output
- Available for AGC
- Enforce Limits
- Participation Factor
  - Used when on participation factor control

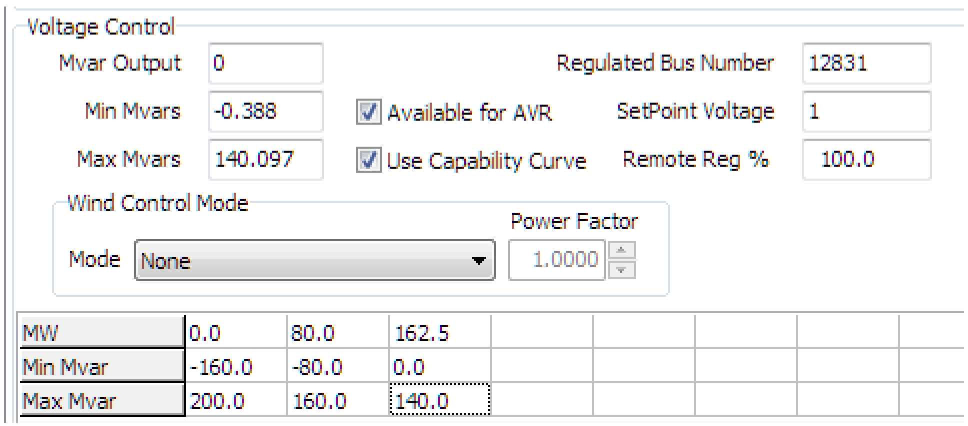
On Generator Dialog



Generator Voltage Control Features (AVR)

- Mvar Output
- Minimum and Maximum MVAR output
  - Or can Use Capability Curve
  - More detailed, but more data
- Voltage Setpoint in per unit
- Available for AVR
- Remote Regulation %
  - Used when more than one generator is controlling the same bus voltage




Voltage Control

Mvar Output	0			Regulated Bus Number	12831
Min Mvars	-0.388	☑	Available for AVR	SetPoint Voltage	1
Max Mvars	140.097	☑	Use Capability Curve	Remote Reg %	100.0

Wind Control Mode

Mode: None — Power Factor: 1.0000

MW	0.0	80.0	162.5
Min Mvar	-160.0	-80.0	0.0
Max Mvar	200.0	160.0	140.0

On Generator Dialog

The Power Flow Equations

- This is the heart of all power system analysis
- Kirchoff's Laws for a power system:
  Sum of the Currents at every Node Equals Zero

Why the “Power Flow”?

- Many loads (like heaters) are just big resistors which should be just an impedance, so why solve the “power flow”?
  - Because our input data is MW and MVAR values. This is because …
  - The experience of utilities for more than 100 years shows that consumers behave similar to constant power over the long-run
  - If the voltage drops on a heater, the power consumption also decreases, but eventually some people get cold and turn up the heat
    - Actually the built in thermostat and control system probably do this automatically

Deriving the Power Flow Equations

- Using the Y-Bus, write Kirchoff's Current Law at every bus as a matrix equation

$$\mathrm{YV} - \mathrm{I}_{\mathrm{generators}} + \mathrm{I}_{\mathrm{loads}} = 0$$

- We are studying POWER systems, so we like to talk about power not current, so

$$\mathrm{S} = \mathrm{VI}^{*} = [\mathrm{V}](\mathrm{V}^{*}\mathrm{Y}^{*}) - [\mathrm{V}]\mathrm{I}^{*}_{\mathrm{generators}} + [\mathrm{V}]\mathrm{I}^{*}_{\mathrm{loads}} = 0$$

$$[\mathrm{V}](\mathrm{V}^{*}\mathrm{Y}^{*}) - \mathrm{S}_{\mathrm{generators}} + \mathrm{S}_{\mathrm{loads}} = 0 \qquad \text{etc.}$$

- These are N-1 complex number equations (where N = the number of buses in the system)

The Power Flow Equations

- These N-1 complex number equations can then be written as 2*(N-1) real number equations as below

$$P_k = 0 = V_k \sum_{m=1}^{N-1} \left[V_m \left[g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m) \right] \right] - P_{Gk} + P_{Lk}$$

Transmission lines, transformers, capacitors

Generators

Loads

$$Q_k = 0 = V_k \sum_{m=1}^{N-1} \left[V_m \left[g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m) \right] \right] - Q_{Gk} + Q_{Lk}$$

Elements of the Y-Bus

Note: The variables g_{km} and b_{km} are the real and imaginary parts of the Y-Bus.
(Y = G + jB or $y_{km} = g_{km} + jb_{km}$)

The Power Flow Equation Variables

- Four parameters describe the bus
  - Voltage magnitude (V)
  - Voltage angle (δ or θ)
  - Real Power Injection ($P = P_{gen} - P_{load}$)
  - Reactive Power Injection ($Q = Q_{gen} - Q_{load}$)
- The objective of the Power Flow algorithm is to determine all four of these values

Three Types of Buses

- In the Power Flow we are given two of these values at each bus and then solve for the other two
- Generally, there are three types of buses in a Power Flow

Type of Bus	Voltage Mag. (V)	Voltage Angle (δ)	Power Injection ($P = P_{gen} – P_{load}$)	Reactive Power Injection ($Q = Q_{gen} – Q_{load}$)
Slack Bus (Vδ-Bus) (only one of these)	GIVEN	GIVEN	SOLVE FOR	SOLVE FOR
PV-Bus (generator on AVR control)	GIVEN	SOLVE FOR	GIVEN	SOLVE FOR
PQ-Bus (load or generator not on AVR control)	SOLVE FOR	SOLVE FOR	GIVEN	GIVEN

How does the Power Flow work?

- The power flow equations are non-linear, which means they can not be directly solved
  - Linear: 35 = 7x - 14, so x = 7
  - Non-linear: 5x = sin(x), so x = ???
- To solve non-linear equations, an iterative technique must be used
- To solve the Power Flow, Newton's Method is normally the best technique
  - Simulator uses this by default.

Solving Non-Linear Equations

- Power Flow Equations are NON-LINEAR equations.
  - There are cos(*) and sin(*) terms which make the equations non-linear
- This means that finding a direct solution to them is impossible
- Thus we must use iterative numerical schemes to determine their solution
- For power flow equations, variations on Newton's Method has been found to be the best technique

Newton's Method

- Discussed in Calculus courses
- Consider a simple scalar equation f(x)
- You can write f(x) as a "Taylor Series"

$$f(x) = f(x_0) + \left(\left. \frac{\partial f}{\partial x} \right|_{x=x_0} \right)(x - x_0) + \frac{1}{2}\left(\left. \frac{\partial^2 f}{\partial x^2} \right|_{x=x_0} \right)(x - x_0)^2 + h.o.t.$$

Newton's Method for Scalar Functions

- Now, approximate this Taylor Series by ignoring all but the first two terms

$$f(x) \approx f(x_0) + \left(\left. \frac{\partial f}{\partial x} \right|_{x=x_0} \right)(x - x_0) \longrightarrow [x - x_0] \approx \left(\left. \frac{\partial f}{\partial x} \right|_{x=x_0} \right)^{-1} [f(x) - f(x_0)]$$

- We are trying to find where f(x) = 0 (the power flow equations sum to zero), therefore we can approximate and estimate

$$x \approx x_0 - \left(\left. \frac{\partial f}{\partial x} \right|_{x=x_0} \right)^{-1} [f(x_0)]$$

Newton's Method

- This is called an iterative method because you take a guess at x_0 and use the "Newton-step" to a new guess called x_1. Then you use x_1 to find x_2, etc.
- Consider $f(x) = x^2-2$. We find

$$\left(\left. \frac{\partial f}{\partial x} \right|_{x=x_k} \right) = 2x_k$$

- Thus $\Delta x_k \approx -\frac{1}{2x_k}(x_k^2 - 2)$ and

$$x_{k+1} \approx x_k + \Delta x_k = x_k + \left[-\frac{1}{2x_k}(x_k^2 - 2) \right]$$

Newton's Method

- Now take and initial guess, say x_0=1, and iterate the previous equation

k	x_k	$f(x_k)$	Δx_k
0	1.0	-1.0	0.5
1	1.5	0.25	-0.08333
2	1.41667	6.953×10^{-3}	-2.454×10^{-3}
3	1.41422	6.024×10^{-6}	Done

- Error decreases quite quickly
  - Quadratic convergence
- $f(x_k)$ is known as the "mismatch"
  - The problem is solved when the mismatch is zero

Non-Linear Equations can have Multiple Solutions

- Consider the previous example.
  - There are two solutions. $x = \sqrt{2}$ and $x = -\sqrt{2}$
- In an iterative scheme, the initial guess determines which solutions you "converge" to. Consider starting at x_0= -1 .

k	x_k	$f(x_k)$	Δx_k
0	-1.0	-1.0	-0.5
1	-1.5	0.25	0.08333
2	-1.41667	6.953×10^{-3}	2.454×10^{-3}
3	-1.41422	6.024×10^{-6}	Done

Converged to $x = -\sqrt{2}$

Non-Linear Equations can have No Real Solution

- Consider the equation $x^2 - \lambda = 0$, where λ is an independent parameter
- The number of real solutions depends on the value of λ .
  - For $\lambda > 0$, there are two real solutions
  - For $\lambda = 0$, there is one real solution at $x=0$
  - For $\lambda < 0$, there are NO real solutions
- Parameter variation can change the number of solutions of nonlinear systems

Newton-Rhapson Convergence Characteristics

- Global convergence characteristics of the N-R algorithm are not easy to characterize
- The N-R algorithm converges quite quickly provided the initial guess is “close enough” to the solution
- However the algorithm doesn’t always converge to the “closest” solution
- Some initial guesses are just plain bad. For example, if $\partial f / \partial x$ is near zero (“ill-conditioned”), the process may diverge

Extending Scalar to Vectors and Matrices

- Newton's method may also be used when you are trying to find a solution where many functions are equal to zero
- Let **f**(x) be a n-dimensional function and let **x** be an n-dimensional vector

$$\mathbf{f}(x) = \begin{bmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Newton's Method for Multiple Equations

- Now, the Newton-step is still defined as

$$\mathbf{x}_{k+1} - \mathbf{x}_k = \Delta \mathbf{x}_k \approx -\left(\left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x_k}} \right)^{-1} \mathbf{f}(\mathbf{x}_k)$$

- But $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ is now a matrix, called the Jacobian

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Solving the Power Flow Equations

- Power Flow Equations are NON-LINEAR equations
  - The cos(*) and sin(*) terms make them non-linear
- This means we have to use an iterative technique to solve them
- Define **f** to be the real and reactive power balance equations at every bus
- Define **x** to be the vector of voltages and angles at every bus (except the slack bus)

Setting up the Power Flow Solution

- Recall

$$P_k = V_k \sum_{m=1}^{N-1} \left[V_m \left[g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m) \right] \right] - P_{Gk} + P_{Lk}$$

$$Q_k = V_k \sum_{m=1}^{N-1} \left[V_m \left[g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m) \right] \right] - Q_{Gk} + Q_{Lk}$$

- Thus,

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_1 \\ Q_1 \\ \hline P_2 \\ Q_2 \\ \hline \vdots \\ \hline P_{n-1} \\ Q_{n-1} \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} \delta_1 \\ V_1 \\ \hline \delta_2 \\ V_2 \\ \hline \vdots \\ \hline \delta_{n-1} \\ V_{n-1} \end{bmatrix}$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \left[\begin{array}{cc|cc|c|cc} \frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial V_1} & \frac{\partial P_1}{\partial \delta_2} & \frac{\partial P_1}{\partial V_2} & \cdots & \frac{\partial P_1}{\partial \delta_{n-1}} & \frac{\partial P_1}{\partial V_{n-1}} \\ \frac{\partial Q_1}{\partial \delta_1} & \frac{\partial Q_1}{\partial V_1} & \frac{\partial Q_1}{\partial \delta_2} & \frac{\partial Q_1}{\partial V_2} & \cdots & \frac{\partial Q_1}{\partial \delta_{n-1}} & \frac{\partial Q_1}{\partial V_{n-1}} \\ \hline \frac{\partial P_2}{\partial \delta_1} & \frac{\partial P_2}{\partial V_1} & \ddots & \ddots & & \vdots & \vdots \\ \frac{\partial Q_2}{\partial \delta_1} & \frac{\partial Q_2}{\partial V_1} & \ddots & \ddots & \ddots & \vdots & \vdots \\ \hline \vdots & \vdots & & \ddots & \ddots & \vdots & \vdots \\ \hline \frac{\partial P_{n-1}}{\partial \delta_1} & \frac{\partial P_{n-1}}{\partial V_1} & \cdots & \cdots & \cdots & \frac{\partial P_{n-1}}{\partial \delta_{n-1}} & \frac{\partial P_{n-1}}{\partial V_{n-1}} \\ \frac{\partial Q_{n-1}}{\partial \delta_1} & \frac{\partial Q_{n-1}}{\partial V_1} & \cdots & \cdots & \cdots & \frac{\partial Q_{n-1}}{\partial \delta_{n-1}} & \frac{\partial Q_{n-1}}{\partial V_{n-1}} \end{array}\right]$$

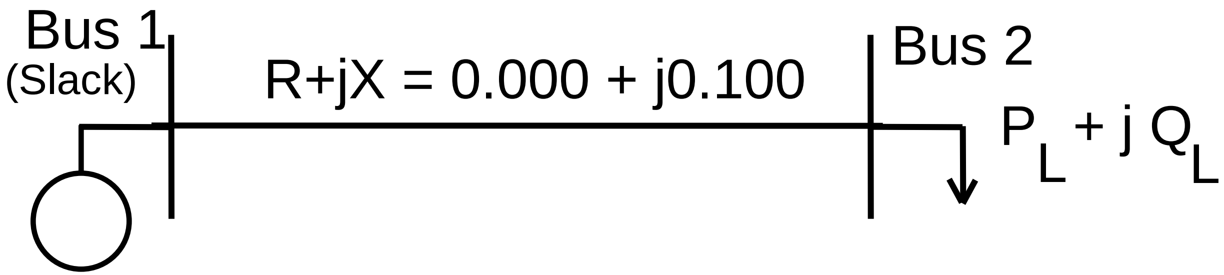
Note: There are n buses, with bus n being the slack bus

Power Flow Solutions

- The power flow equations exhibit the same behavior as other non-linear equations
- In theory there are up to 2^{n-1} solutions to the power flow equations
  - Of these, ALL or NONE may be real number solutions
- In power system analysis, we are normally interested only in the solution that is the "high voltage" solution for every bus

Example Two-Bus Power System

- Consider this lossless (R=0) example system



- Variable Definitions

$$\theta = \text{Voltage Angle at Bus 2}$$

$$V = \text{Voltage Magnitude at Bus 2}$$

$$B = \left(\frac{-\mathrm{x}}{\mathrm{r}^2 + \mathrm{x}^2}\right) = \left(\frac{-0.1}{0 + 0.1^2}\right) = -10$$

- Power balance equations :

$$PMismatch = P_L - BV\sin(\theta)$$

$$QMismatch = Q_L + BV\cos(\theta) - BV^2$$

How do power flow solutions vary as load is changed?

- Solution: Calculate a series of power flow solutions for various load levels
- First determine the following

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \mathbf{J}(\theta, V) = \begin{bmatrix} -BV\cos(\theta) & -B\sin(\theta) \\ -BV\sin(\theta) & B\cos(\theta) - 2BV \end{bmatrix}$$

$$\mathbf{x}_k = \begin{bmatrix} \theta \\ V \end{bmatrix} \quad ; \quad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_L - BV\sin(\theta) \\ Q_L + BV\cos(\theta) - BV^2 \end{bmatrix}$$

Example Power Flow Solution

- Now for each load level, iterate from an initial guess until the solution (hopefully) converges

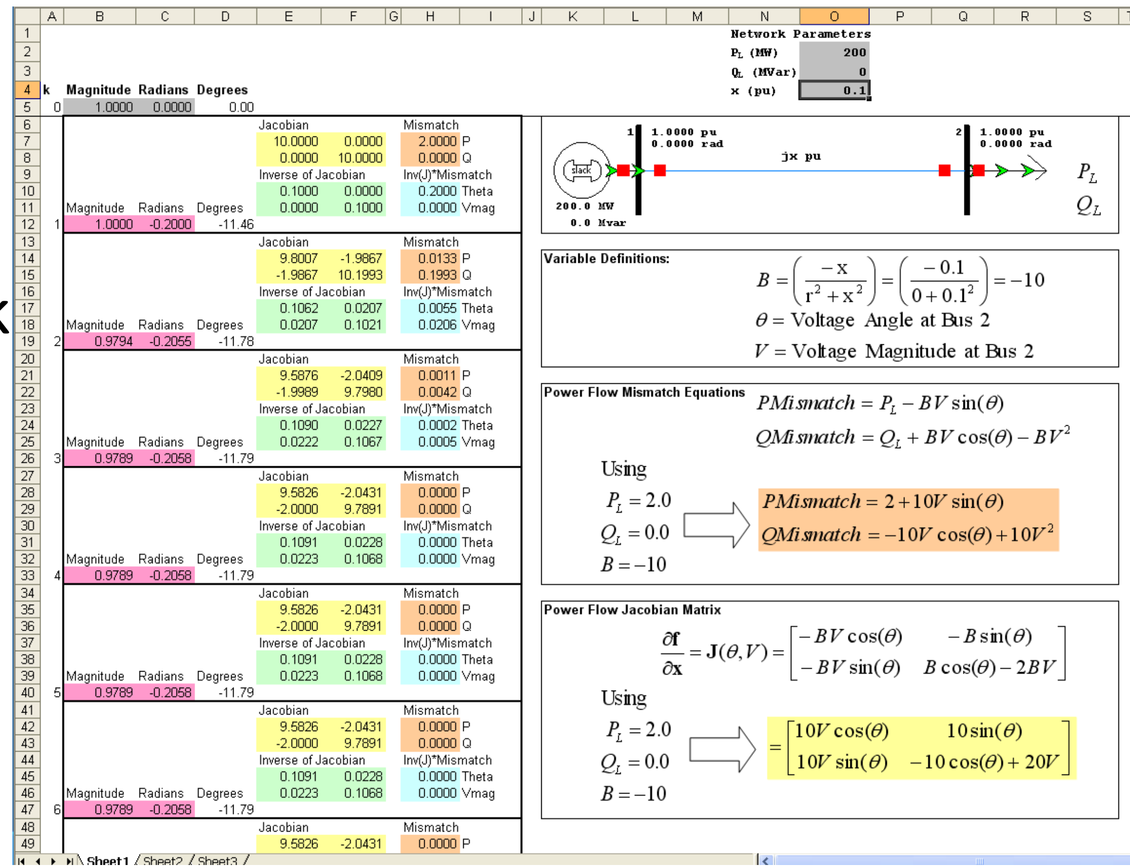
$$x \approx x_0 - \left(\left. \frac{\partial f}{\partial x} \right|_{x=x_0} \right)^{-1} \left[f(x_0) \right]$$

- Consider a per unit load of P_L = 2 pu and Q_L=0 pu
  - A 200 MW load since SBase = 100 MVA
- Then consider a "flat start" for the initial guess: $V_0 = 1$; $\theta_0 = 0$.
- Also consider a start of $V_0 = 0.5$; $\theta_0 = -1$.

49

Simple 2-Bus Power Flow in Excel

- Open Microsoft Excel
  - …\S01_SystemModeling\Power Flow Two Bus.xls
  - Gray Shaded cells are user inputs (initial guess, load and network parameters)



Iteration Results for Power Flow Solution

- Change Initial Voltage to $V_0 = 1$; $\theta_0 = 0$
  - Cell B5 = 1.0
  - Cell C5 = 0.0

k	V^k	θ^k (radians)
0	1.0000	-0.0000
1	1.0000	-0.2000
2	0.9794	-0.2055
3	0.9789	-0.2058
4	0.9789	-0.2058

High Voltage Solution

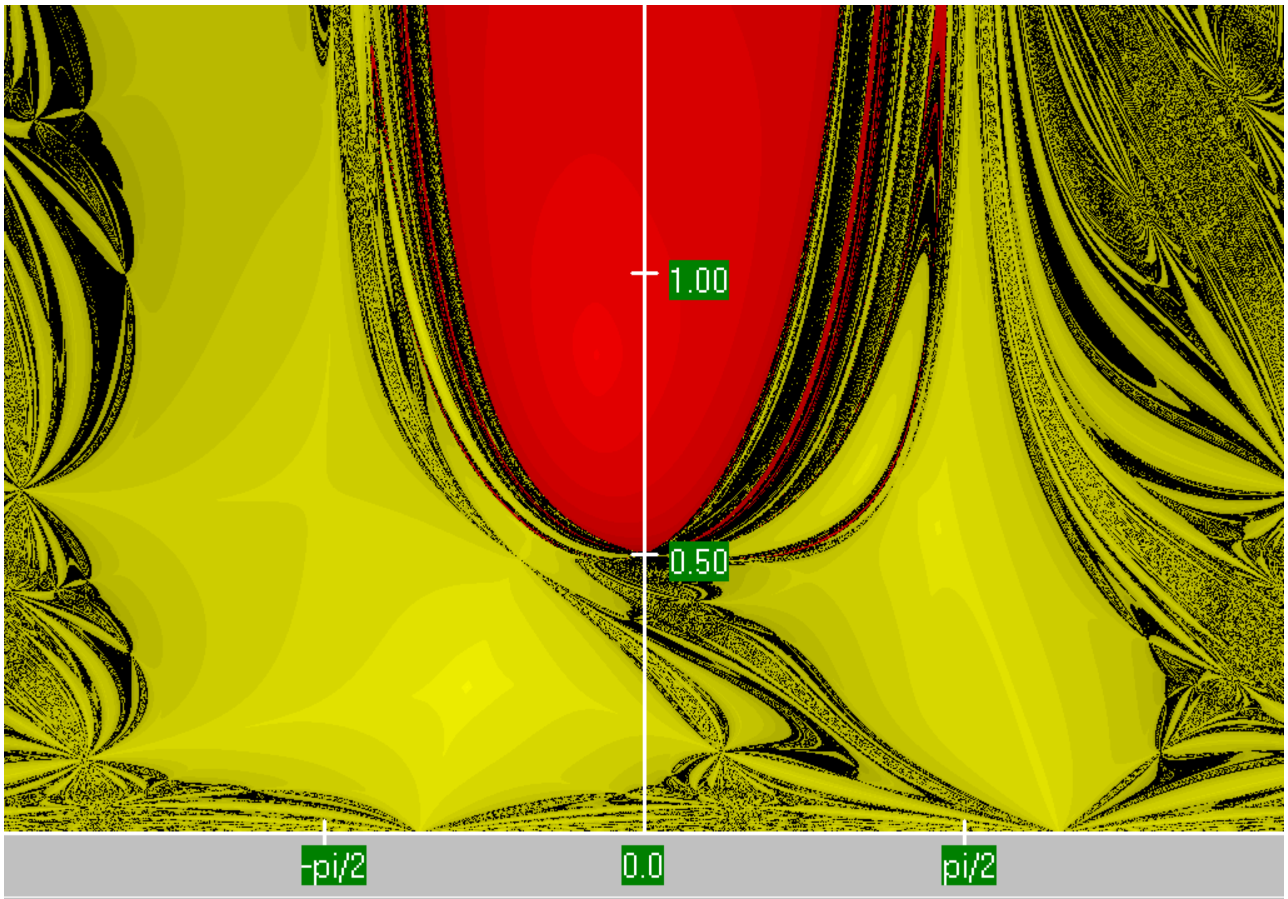
- Change Initial Voltage to $V_0 = 0.5$; $\theta_0 = -1$
  - Cell B5 = 0.5
  - Cell C5 = -1.0

k	V^k	θ^k (radians)
0	0.5000	-1.0000
1	0.0723	-1.5152
2	0.2010	-1.3397
3	0.2042	-1.3657
4	0.2043	-1.3650

Low Voltage Solution

Region of Convergence

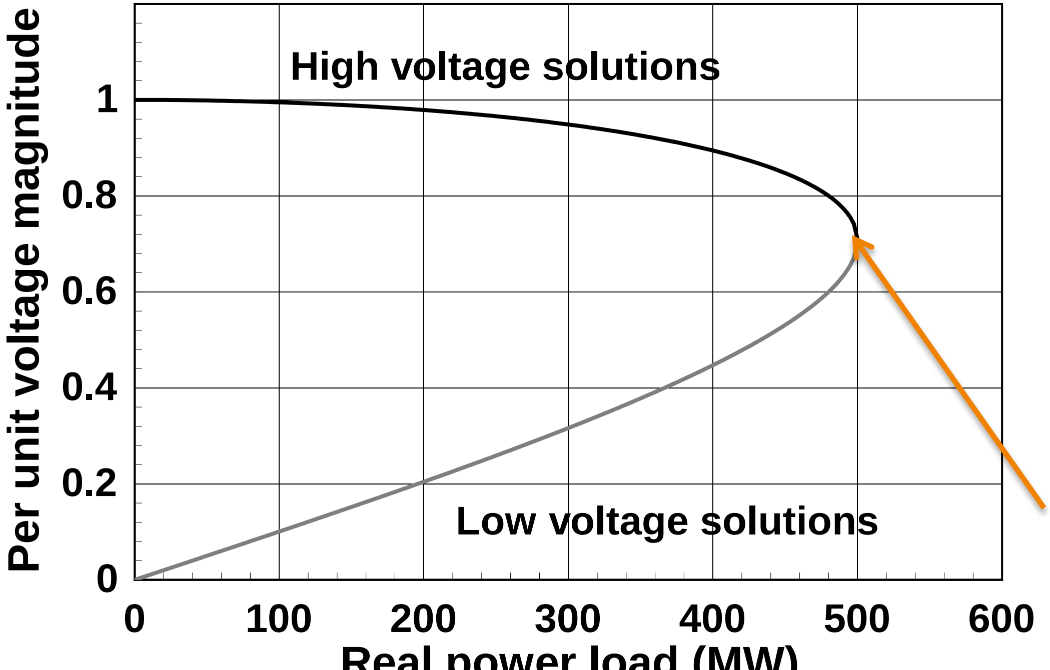
Initial guess of Bus 2 angle is plotted on x-axis, voltage magnitude on y-axis, for load = 200 MW, 100 Mvar; x = 0.1 pu



Initial guesses in the red region converge to the high voltage solution, while those in the yellow region converge to the low voltage solution

PV-Curves

- A PV-Curve is generated by plotting these two solutions as the real power load is varied with the reactive load and impedance constant
- Experiment with parameters in cells O2-O4
- Higher transmission impedance usually decreases the system capacity (maximum load)

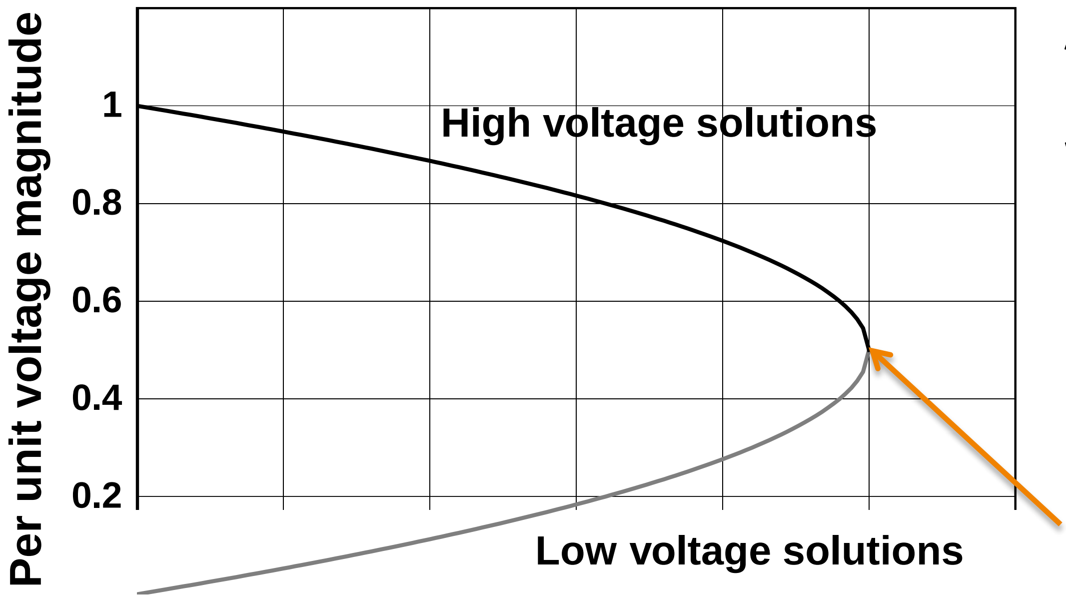


Note: load values below 500 have two solutions, those above 500 have no solution

For a critical value (i.e. the "nose point") there is only one solution

QV-Curves

- A QV-Curve is generated by plotting these two solutions as the reactive power load is varied with the real load and impedance constant



Again: load values below 250 have two solutions, while those above 250 have no solution

For a critical value (i.e. the "nose point") there is only one solution

Maximum Loadability

- The “nose points” on the PV and QV curves are critical values
- They correspond to points of maximum loadability from a static point of view
  - Note: this only considers the static equations. Other problems can occur long before the power system reaches this points: lines overheating or system dynamic problems
- The critical values depend on the relative allocation between real and reactive power

Mathematical Characteristics of the Critical Value

- At the critical value, the power flow Jacobian will be singular
  - A singular matrix is one that has a zero determinant (a generalization of scalar case when df/dx = 0)
- A singular matrix CANNOT be inverted
  - Thus the Newton power flow does not work well near this point because it requires us to invert the Jacobian matrix

$$\mathbf{x}_{k+1} - \mathbf{x}_k = \Delta \mathbf{x}_k \approx -\left(\left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x_k}} \right)^{-1} \mathbf{f}(\mathbf{x}_k)$$