All About Power Flow Alternative Solutions Including How to Avoid Them

Thomas J. Overbye
O’Donnell Foundation Chair III
Texas A&M University
overbye@tamu.edu

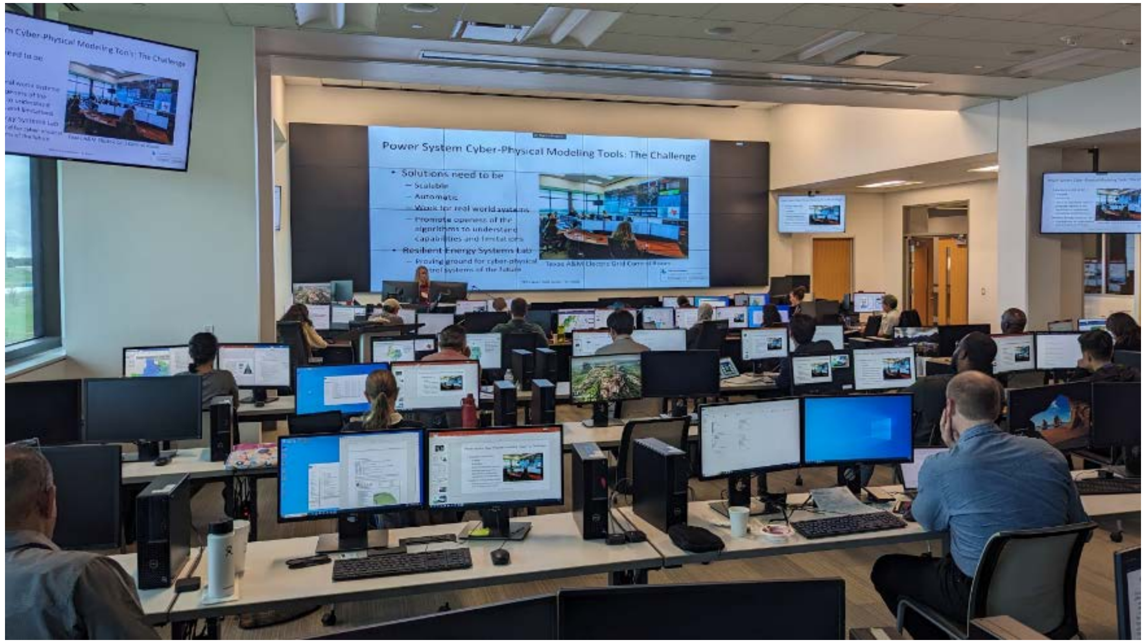
Texas A&M Smart Grid Center Webinar
September 3, 2025

Overview

- The power flow, probably the most common power system analysis tool, is used to determine the bus voltages, which in turn are used to get all the power flows.
- In its standard form (positive sequence, ac) the power flow involves solving sets of nonlinear equations; as such, these equations often have multiple solutions, or may have no solution
  - If solutions exist, almost always one is the desired solution that corresponds to how the grid would actually operate; it will be referred to as the "operable solution" (OprS)
  - Any other solutions will be referred to as an "alternative solutions" (AltSs)
- The focus of this presentation of this presentation is on these alternative solutions, including how to detect them and avoid them.
- These slides are available at **overbye.engr.tamu.edu/presentations/**

Aside: Texas A&M SGC Electric Power Short Courses for Fall 2025

- Building on its success from prior semesters, the Texas A&M Smart Grid Center (SGC) will be offering at least three more short courses in Fall 2025 at our control lab/training facility in Bryan TX; more information is at **epg.engr.tamu.edu/short-courses-main/**
  - **Primer on the Planning and Operation of Large-Scale Electric Grids** (September 9 to 11)
  - **Fundamental of Electric Transmission Planning** (October 7 to 9)
  - **Introduction to Artificial Intelligence in Power Systems** (November 4 to 6)



Presentation Abstract

- The presentation covers alternative power flow solutions (AltSs).
- While AltSs are thought to be rare, they do occur, and can substantially change a solution. However, there can be many AltSs, and some might not impact the solution in the area of interest.
  - While usually good, this also means the presence of the AltS can be easily missed.
- Avoiding AltS involves a combination of
  - Utilizing available options to modify the power flow solution (Newton-Raphson here)
  - Avoiding, as much as possible, large power flow changes; several smaller changes might be a better approach
  - When new buses are manually added setting their initial angles close to the existing angle value
  - Effectively managing the explicitly modeled wye-delta phase shifts

AltSs in Practice

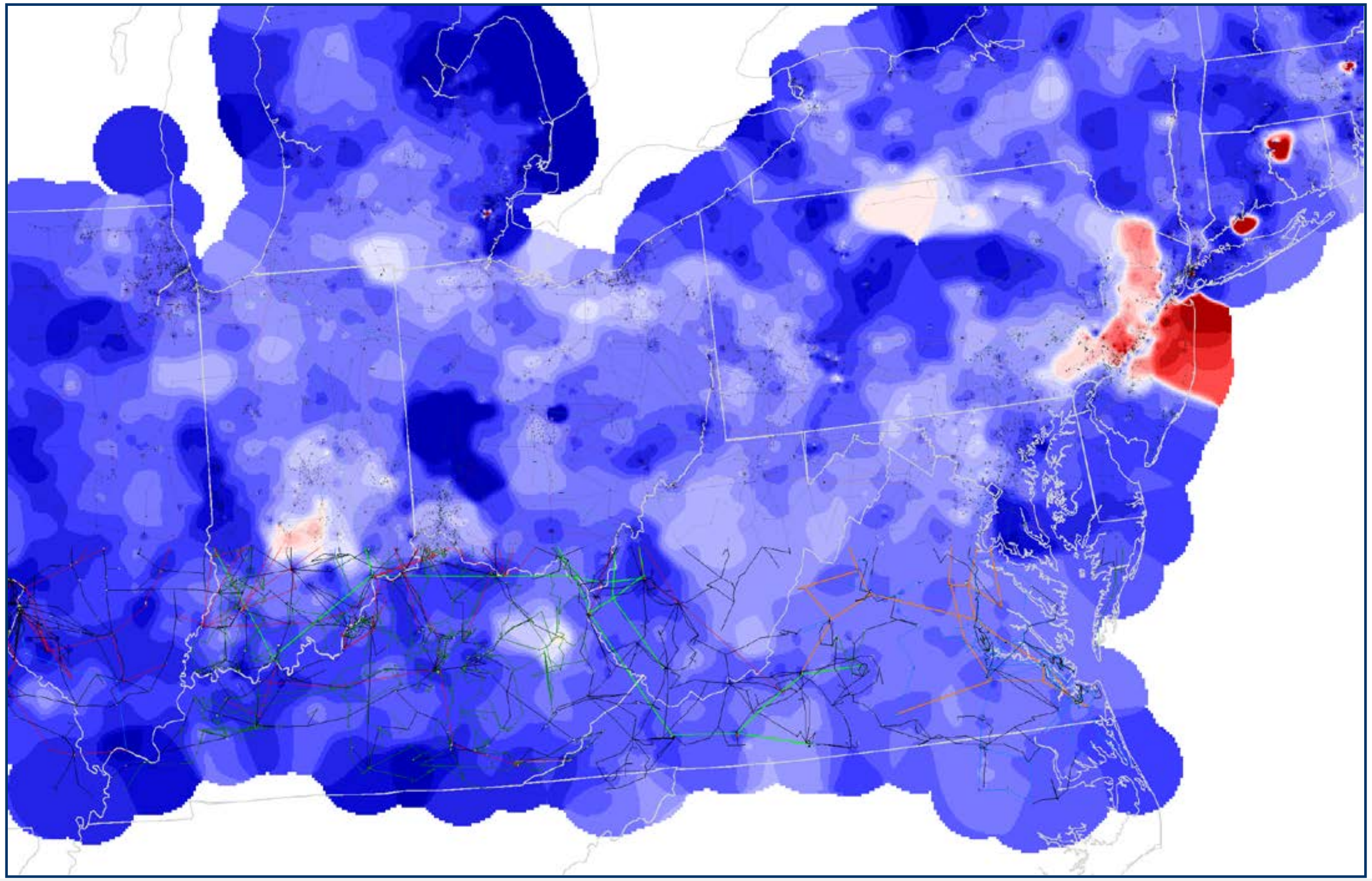


Image shows planning study voltage contour (with red low) that had 65 energized 115,138, or 230 kV buses with voltages below 0.90 pu.

The lowest 138 kV voltage was 0.836 pu; lowest 34.5 kV was 0.621 pu; case contained 42,766 buses; once a power flow solves to an alternative solution, it tends to stay there fore subsequent changes!

Power Flow versus Load Flow

- The power flow is one of the most widely used power system analysis tools
  - The terms "power flow" and "load flow" are used synonymously; I prefer the term power flow since the power "flows", not the load
  - But both are commonly used, and have been commonly used in the power industry for at least 60 years
- The classic paper in this area is by W.F. Tinney and C.E. Hart, "Power Flow Solution by Newton's Method," *IEEE Power App System*, Nov 1967
  - The power flow usage is not new, with a 1956 paper by Ward and Hale titled "Digital Computer Solution of Power-Flow Problems"
  - In that 1956 paper there are a number of "load flow" references; a nice history of the power flow is given in an insert by F. Alvarado and R. Thomas in T.J. Overbye, J.D. Weber, "Visualizing the Electric Grid," *IEEE Spectrum*, Feb 2001

Modeling Cautions

- "All models are wrong but some are useful," George Box, *Empirical Model-Building and Response Surfaces*, (1987, p. 424)
  - Models are an approximation to reality, not reality, so they always have some degree of approximation
  - Box went on to say that the practical question is how wrong to they have to be to not be useful
- A good part of engineering is deciding what is the appropriate level of modeling, and knowing under what conditions the model will fail.
- Of course, engineering often involves using widely accepted modeling and analysis approaches for common problems.
- Models "failing" is particularly germane when talking about power flow AltSs.

Linear versus Nonlinear Systems

A function **H** is linear if

$$\mathbf{H}(\alpha_1 \boldsymbol{\mu}_1 + \alpha_2 \boldsymbol{\mu}_2) = \alpha_1 \mathbf{H}(\boldsymbol{\mu}_1) + \alpha_2 \mathbf{H}(\boldsymbol{\mu}_2)$$

That is

1) the output is proportional to the input
2) the principle of superposition holds

Linear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = \mathbf{A}\,\mathbf{x}$

$$\mathbf{y} = \mathbf{A}\,(\mathbf{x}_1 + \mathbf{x}_2) = \mathbf{A}\,\mathbf{x}_1 + \mathbf{A}\,\mathbf{x}_2$$

Nonlinear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}^2$

$$\mathbf{y} = c(\mathbf{x}_1 + \mathbf{x}_2)^2 \neq (c\mathbf{x}_1)^2 + (c\,\mathbf{x}_2)^2$$

Linear systems have a single solution.

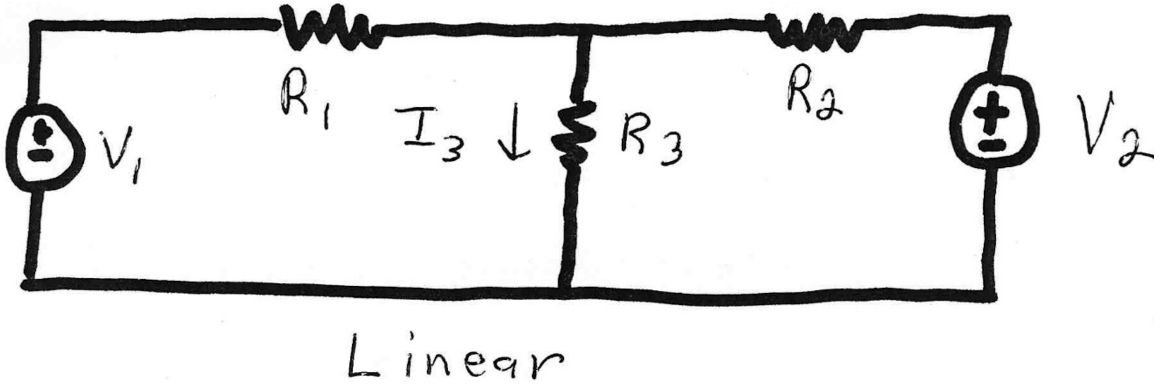
The dc power flow, common in power market analysis, involves solving a set of linear equations. Hence it has the nice property of always have a single solution.

Linear Power System Elements

- Resistors, inductors, capacitors, independent voltage and current sources are linear circuit elements

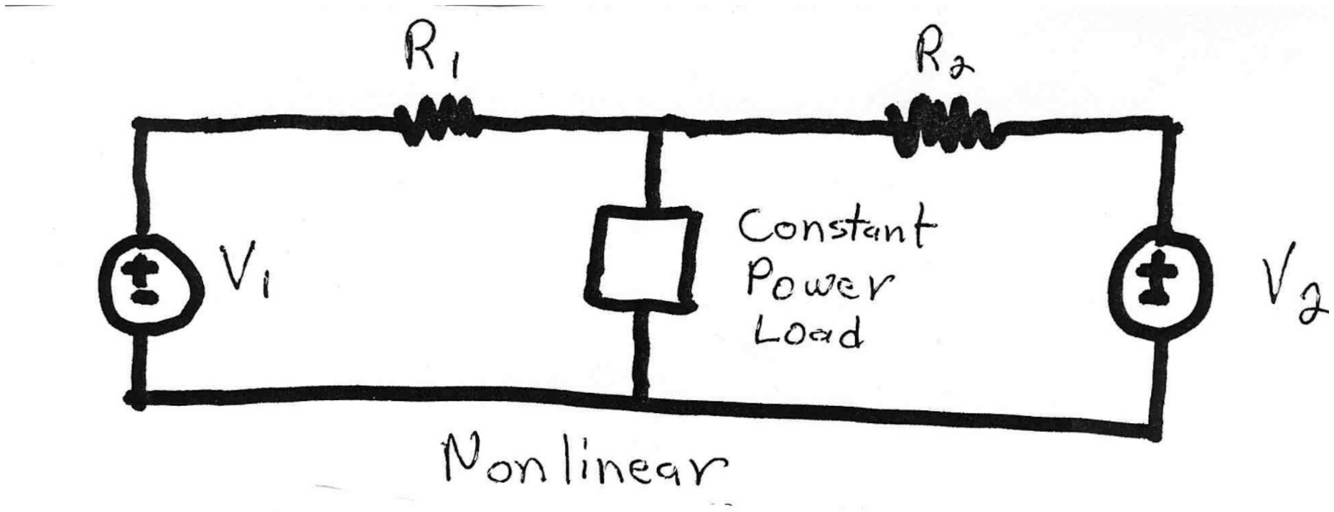
  $V = R\ I, \quad V = j\omega L\ I, V = 1/(j\omega C)\ I$

- Such systems may be analyzed by superposition



Mixed Linear and Nonlinear Models

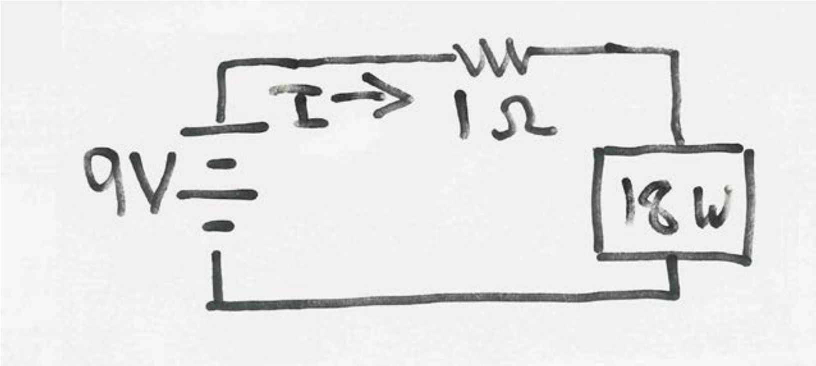
- Constant power loads and generator injections are nonlinear and hence systems with these elements can not be analyzed by superposition



Nonlinear problems can be very difficult to solve, and usually require an iterative approach

Multiple Solution DC Circuit Example

- The dc system shown below has two solutions



Assumed the 18 watt box is a resistive load.

If the load was zero power, then there are also two solutions: 1) open circuit, and 2) short circuit.

The equation we're solving is

$$I^2 R_{Load} = \left(\frac{9 \text{ volts}}{1\Omega + R_{Load}}\right)^2 R_{Load} = 18 \text{ watts}$$

One solution is $R_{Load} = 2\Omega$

Other solution is $R_{Load} = 0.5\Omega$

The maximum load is 20.25 W.

Power Flow: Solving the Power Balance Equations

- The power flow involves solving the nonlinear power balance equations at every bus in a system (except for the slack bus)

$$S_i = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$P_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

These equations are formulated with the voltages in polar notation; they can also be formulated with the voltages in rectangular notation ($V_i = e_i + jf_i$). Of course, they give the same solution.

Slack (or Reference) Bus

- We can not arbitrarily specify **S** at all buses because total generation must equal total load + total losses.
- We also need an angle reference bus.
- To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.
- There is no slack bus in a real system; the frequency changes locally when the power supplied does not match the power consumed.

Newton-Raphson Algorithm

- Most common technique for solving the power flow problem is to use the Newton-Raphson (N-R) algorithm. Key idea behind Newton-Raphson is to use sequential linearization with

  General form of problem: Find an $\mathbf{x}$ such that $\mathbf{f}(\hat{\mathbf{x}}) = 0$

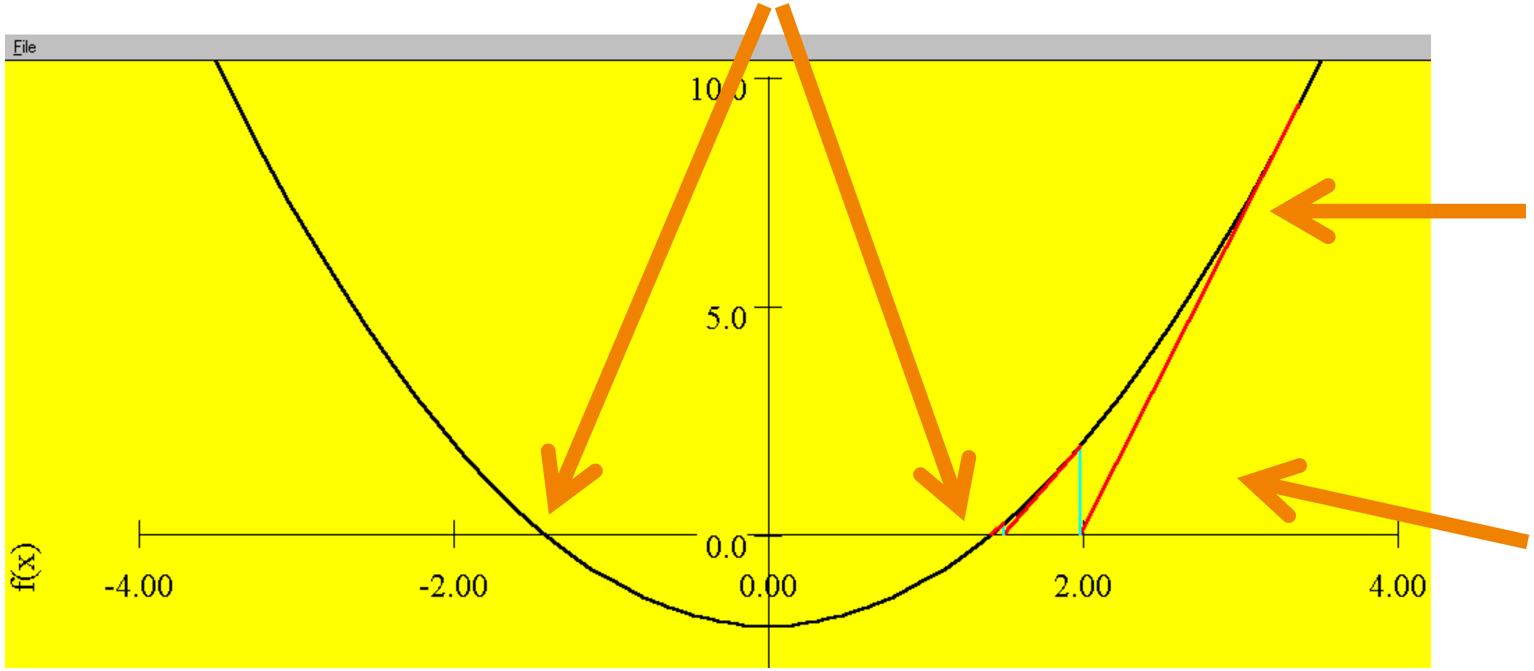
- For a scalar function the sequential iteration is

1. Make an initial guess for $\hat{x}$, $x^{(0)}$
2. Set the iteration counter v to zero, and iteratively solve for a new estimate of $\hat{x}$, $x^{(v+1)}$, until $f(x^{(v+1)})$ is below a tolerance

$$x^{(v+1)} = x^{(v)} - \left[\frac{df(x^{(v)})}{dx}\right]^{-1} f(x^{(v)})$$

Newton-Raphson (N-R) to Solve $x^2 - 2 = 0$

- The solution values are where the function is equal to zero; here there are two: square root of 2 and minus square root of 2.
- The initial guesses that result in convergence of the N-R to a particular solution are known as the solutions region of attraction (ROA).



At each iteration the N-R method uses a linear approximation to determine the next value for x.

Power Flow Variables

Assume the slack bus is the first bus (with a fixed voltage angle/magnitude).

We then need to determine the voltage angle/magnitude at the other buses.

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \\ |V_2| \\ \vdots \\ |V_n| \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ \vdots \\ P_n(\mathbf{x}) - P_{Gn} + P_{Dn} \\ Q_2(\mathbf{x}) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(\mathbf{x}) - Q_{Gn} + Q_{Dn} \end{bmatrix}$$

Newton-Raphson Power Flow Solution

The power flow is solved using the same procedure discussed with the general Newton-Raphson:

Set $v = 0$; make an initial guess of $\mathbf{x}$, $\mathbf{x}^{(v)}$

While $\left\| \mathbf{f}(\mathbf{x}^{(v)}) \right\| > \varepsilon$ Do

$$\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$$

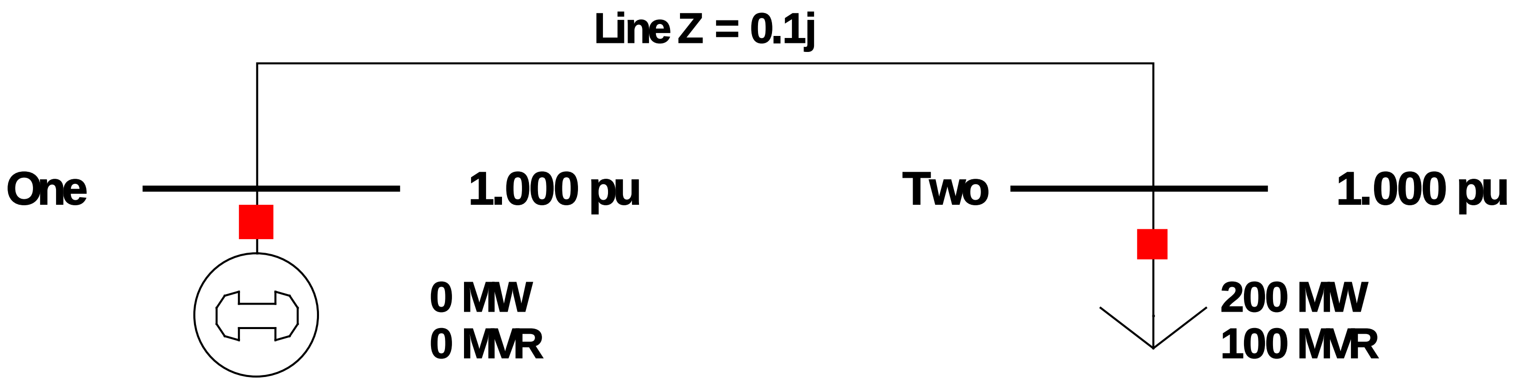
$$v = v + 1$$

End While

Commercial NR algorithms have many additional complications, including those associated with managing the voltages and reactive powers, and managing the area real power interchanges.

Two Bus Newton-Raphson Example

- For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and S_{Base} = 100 MVA.



$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix} \qquad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

Two Bus Example, cont'd

General power balance equations

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

The rectangular form

$$P_i = \sum_{j=1}^{n} \{ e_i (e_j G_{ij} - f_j B_{ij}) + f_i (f_j G_{ij} + e_j B_{ij}) \} \quad \text{(B-1)}$$

$$Q_i = \sum_{j=1}^{n} \{ f_i (e_j G_{ij} - f_j B_{ij}) - e_i (f_j G_{ij} + e_j B_{ij}) \} \quad \text{(B-2)}$$

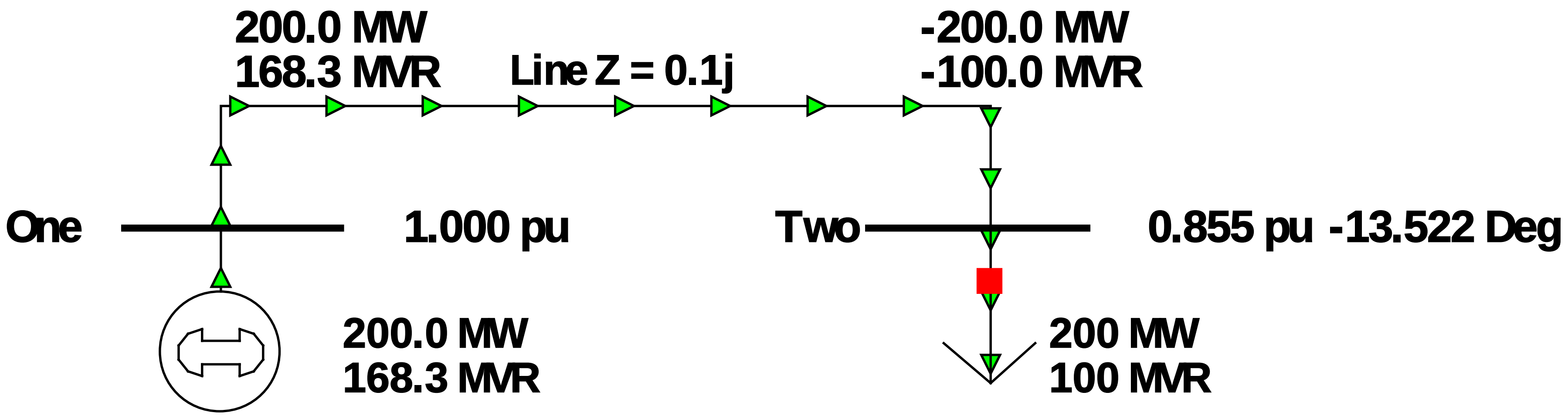
Bus two power balance equations

$$|V_2||V_1|(10\sin\theta_2) + 2.0 = 0$$

$$|V_2||V_1|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 = 0$$

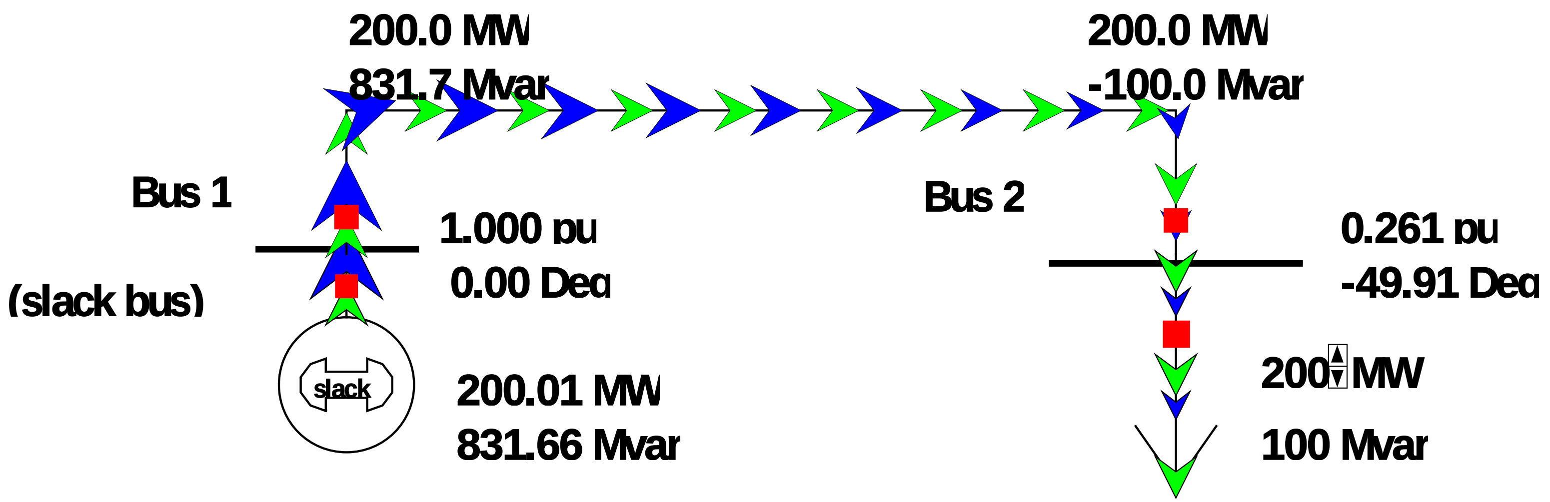
Two Bus Solved Values

- The power flow then determines the Bus 2 values (voltage magnitude and angle). Once these are known we can calculate all the other system values, such as the line flows and the generator reactive power.



Alternative Power Flow Solutions

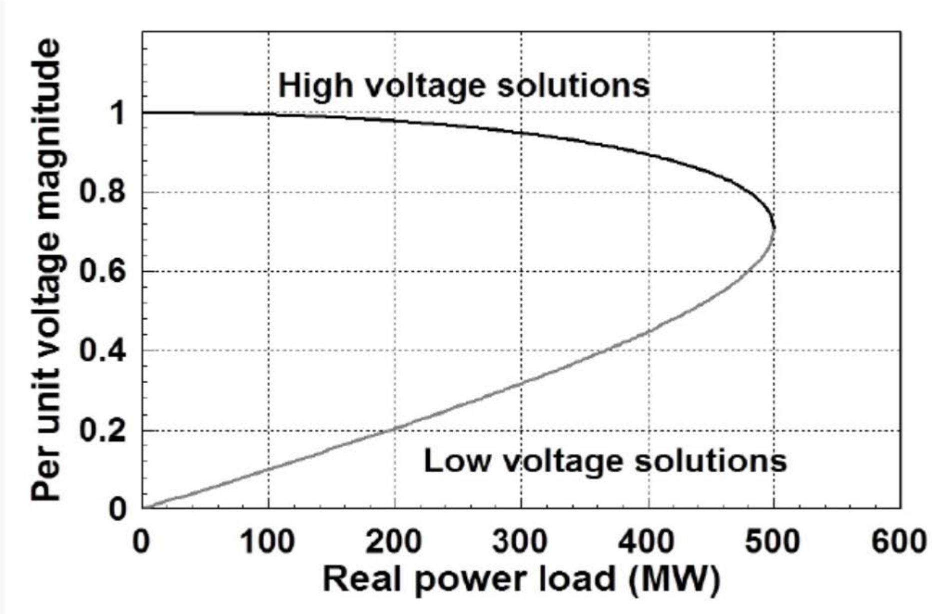
- If the load is assumed to be truly constant power, then the power flow equations actually have two solutions!

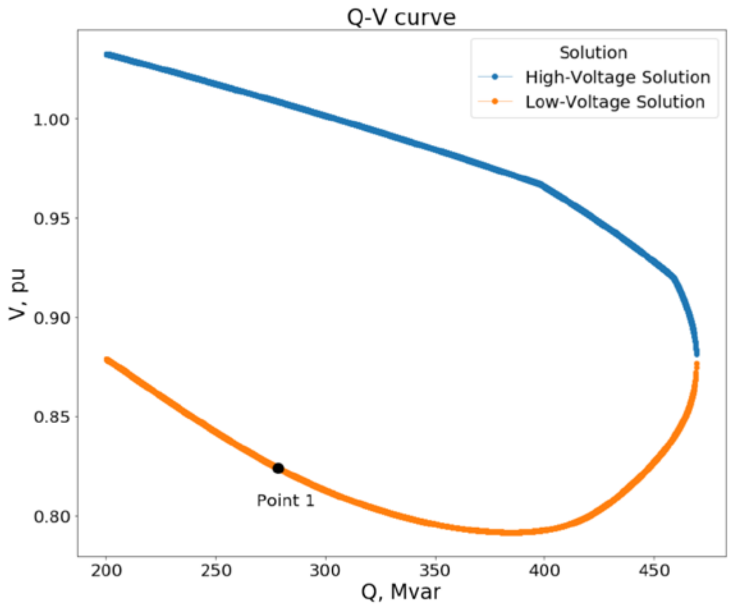


This second solution does not represent an actual operable solution, but it does satisfy the power flow equations. This solution is denoted here as an alternative solution (AltS); these solutions are also sometimes called low voltage solutions since some voltages in AltSs are often (but not always!) low.

PV Curves (Nose Curves) and Parameter Space

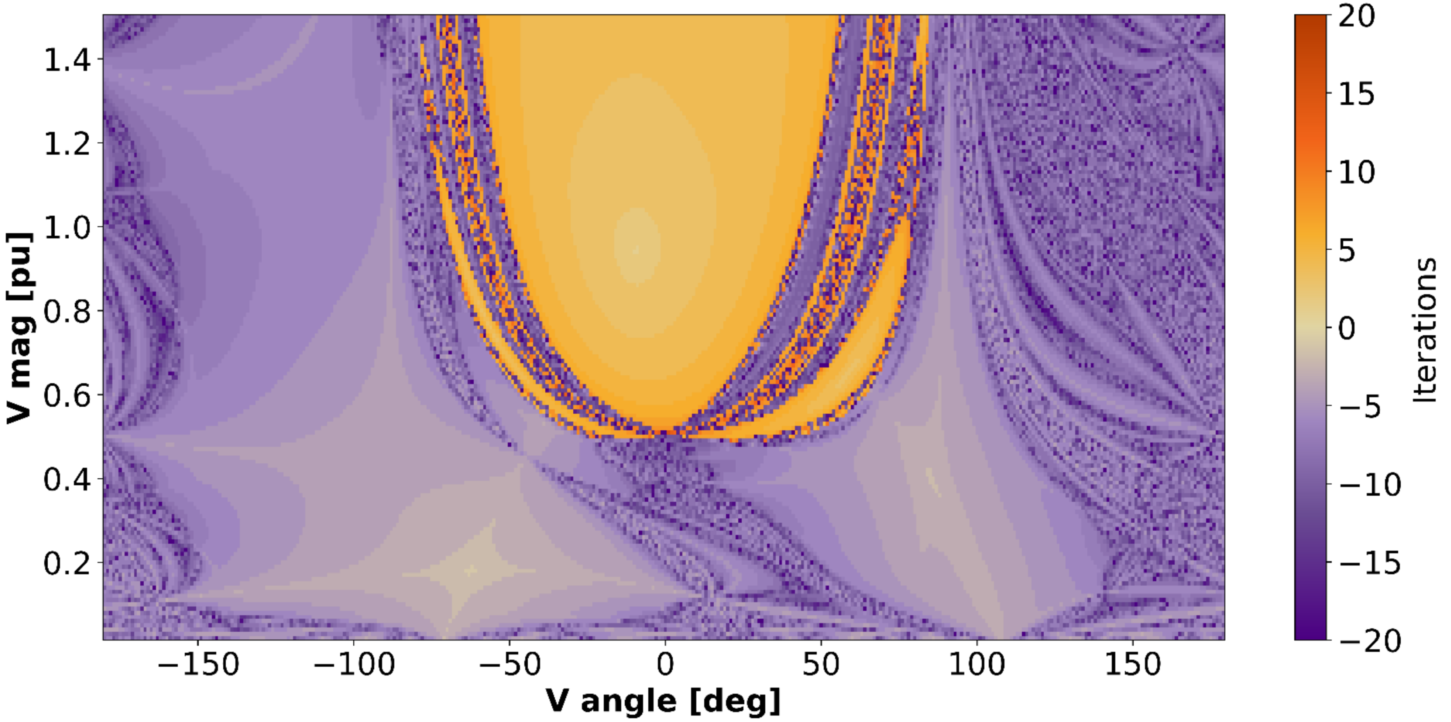
- PV and QV curves (sometimes known as “Nose Curves”) plot the variation of the solution voltages as a set of parameters are varied (e.g., real power for a PV curve, reactive power for a QV curve).
- The left image shows a standard PV curve, whereas the right image shows a QV curve example where an AltS occurs at a relatively high voltage.





Two Bus Region of Attraction

- The region of attraction (ROA, also called domain of attraction) for the NR algorithm is the set of voltage guesses that converge to a particular solution.
- Image shows the ROA for different initial guesses of Bus 2 angle (x-axis) and magnitude (y-axis), with negative iterations for AltS convergence.

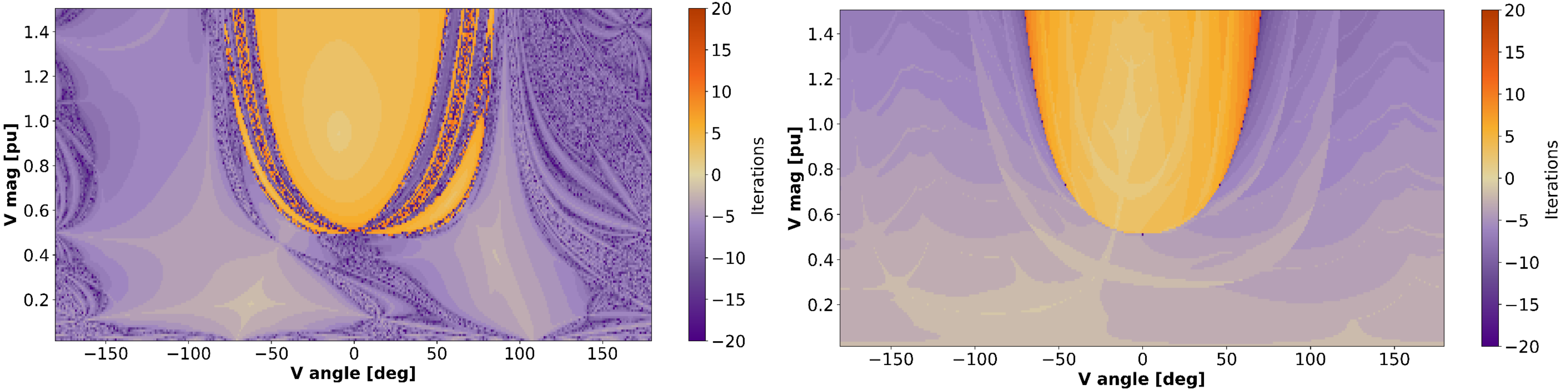


Orange region initial guesses converge to the OprS, while the purple region guesses converges to an AltS, here with a much lower voltage.

Note that initial guesses with one per unit voltage can result in convergence to the AltS.

The ROA Depends on the Solution Algorithm

- The ROA does depend on the algorithm, with the left image showing the ROA for the polar NR, and the right image the ROA for the rectangular NR using the optimal multiplier approach [a] to prevent divergence.

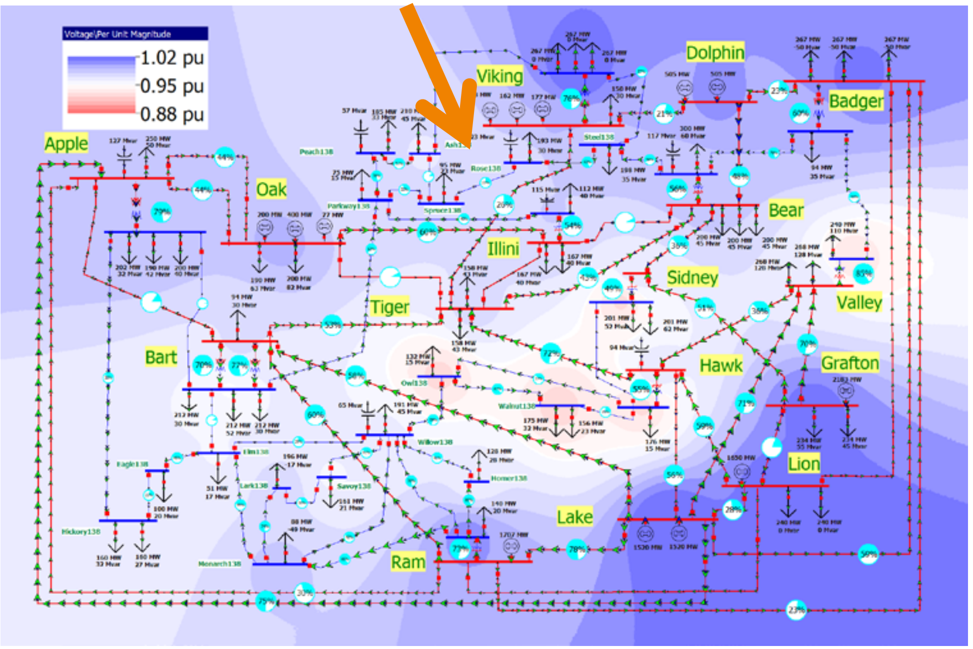


[a] S. Iwamoto and Y. Tamura, "A Load Flow Calculation Method for Ill-Conditioned Power Systems," *IEEE Trans. Power App. & Syst.* vol. PAS-100, no. 4, pp. 1736-1743, April 1981.

Images source: K. Zhgun, S. Kunkolienkar, F. Safdarian, T.J. Overbye, J. Weber, “Improving Power Flow Convergence By Leveraging Region of Attraction Visualization,” 2025 North American Power Symposium (NAPS), Hartford, CT, October 2025.

The ROA Might Be Relatively Small

- The below left image shows the OprS voltage contour for a 42-bus system, and the right image the ROA for an initial voltage guess at Bus 34 (Rose138) with all other voltages set to their OprS values; note that flat start voltage (1∠0°) is not in the ROA for the OprS.



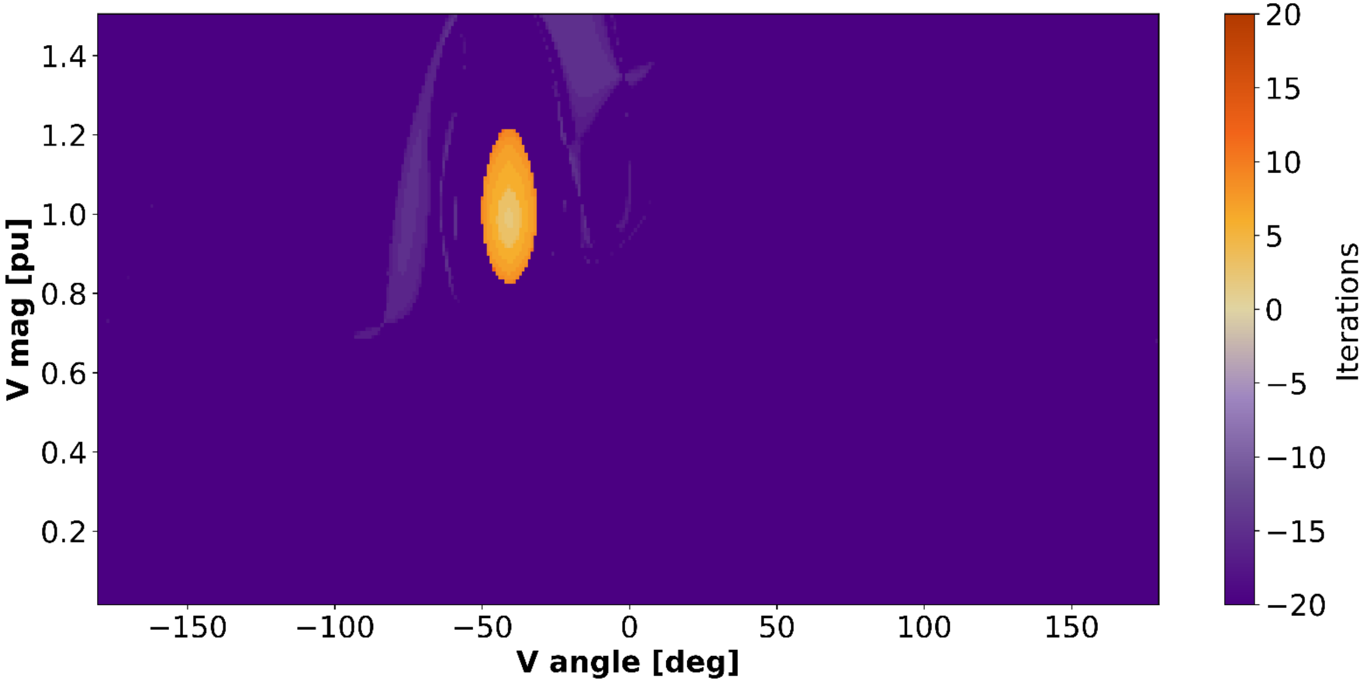
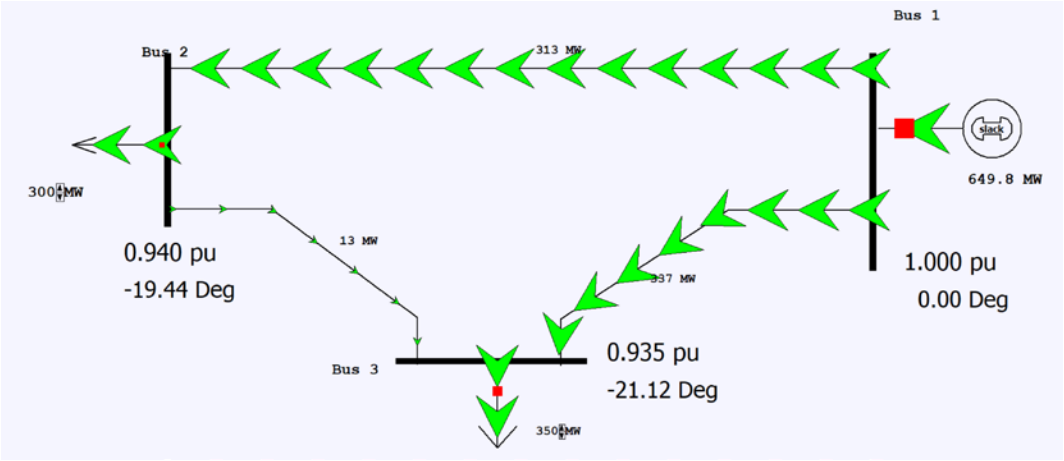
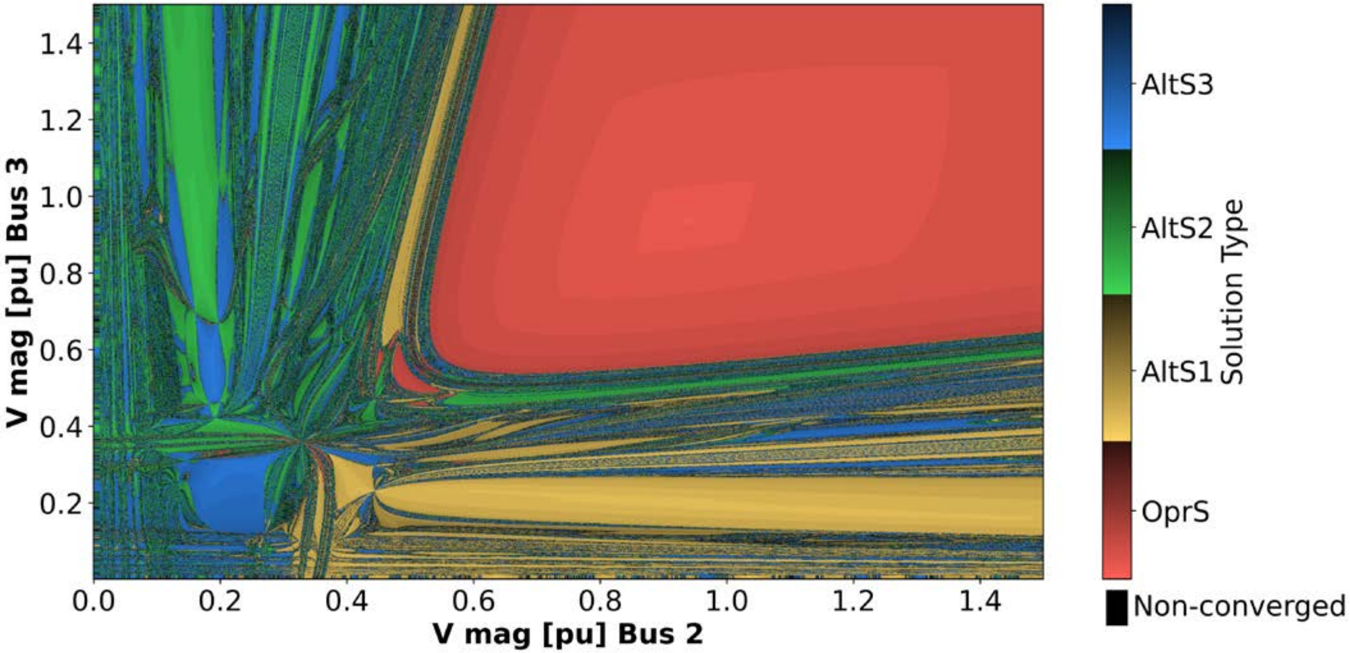


Image source: K. Zhgun, S. Kunkolienkar, F. Safdarian, T.J. Overbye, J. Weber, "Improving Power Flow Convergence By Leveraging Region of Attraction Visualization," 2025 North American Power Symposium (NAPS), Hartford, CT, October 2025.

ROAs Showing Values for Multiple Buses

- The dimension of the ROA is two times the number of non-slack buses; hence only a low dimension projection (e.g. 2D) is possible to show.
- Common projections are 1) voltage magnitude and angle at one bus, 2) voltage magnitudes at two buses, or 3) voltage angles at two buses.

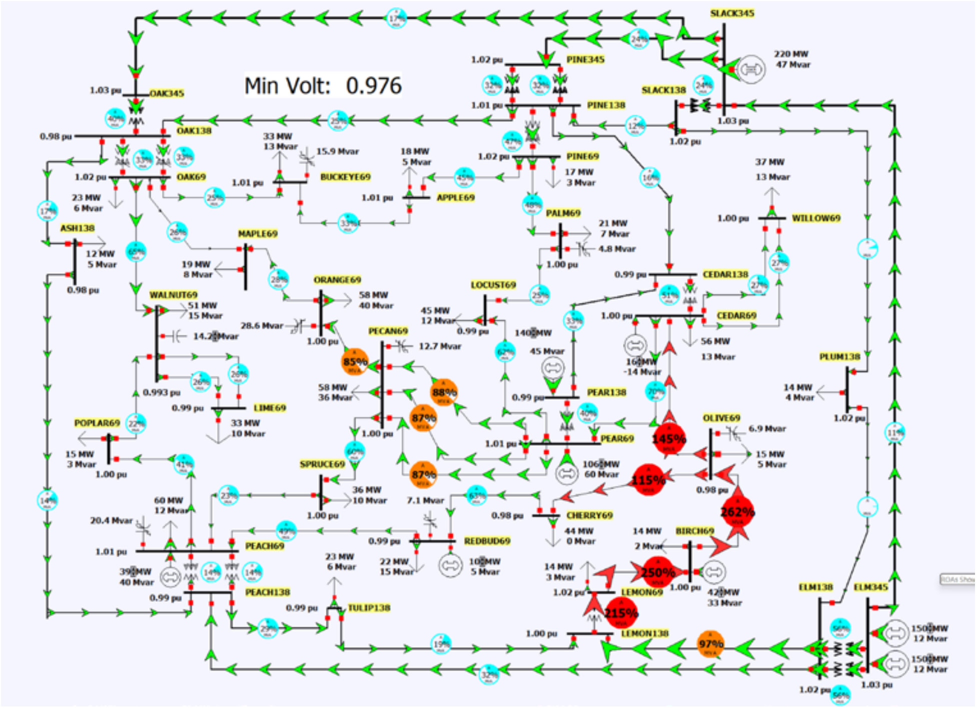


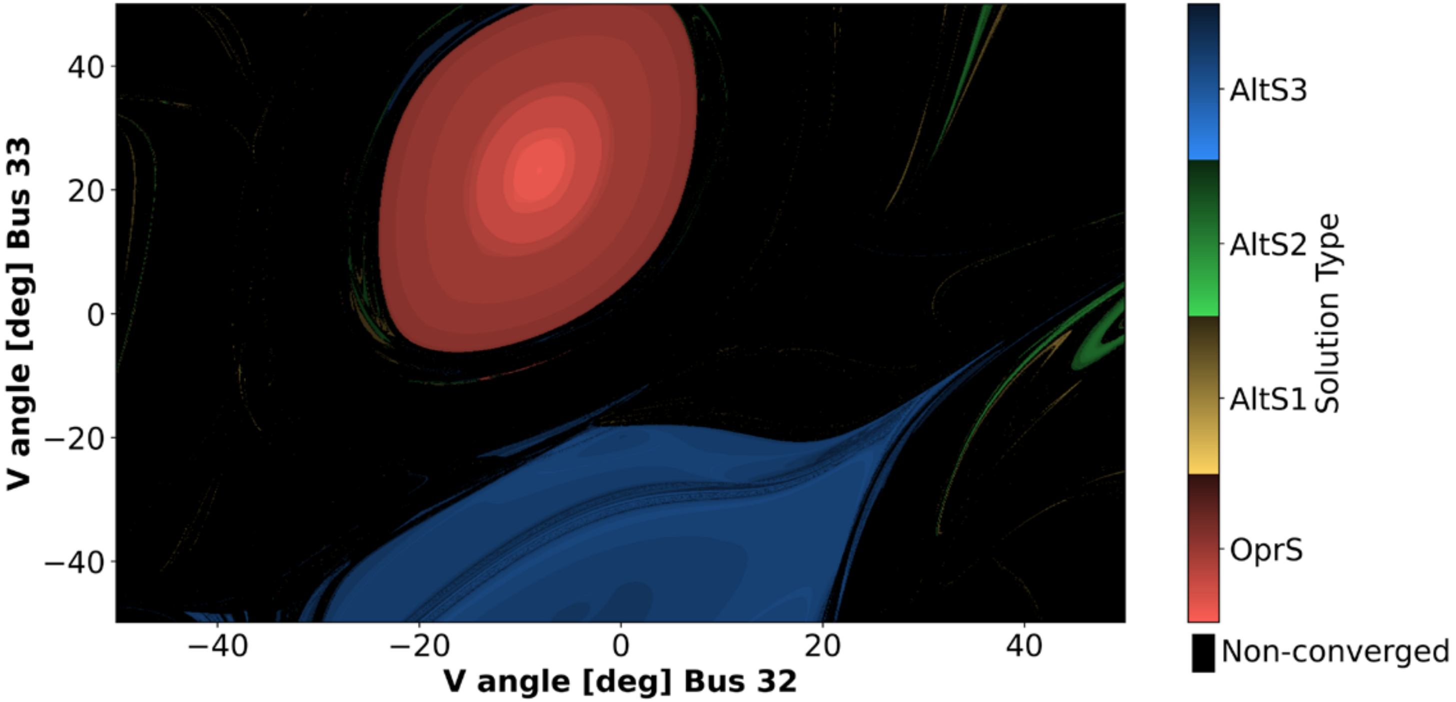


Thanks to TAMU graduate student Sanjana Kunkolienkar for the right image, which required more than 2 million power flow solutions to produce!

ROAs Showing Values for Multiple Buses: Angles

- This example shows the ROA for phase shifter terminal bus angles (flat start for all others), with four solutions found. Here the full flat start solution (zero degrees at Buses 32 and 33) is not in the ROA for the OprS.





Again thanks to TAMU graduate student Sanjana Kunkolienkar for the right image, which here required 1 million power flow solutions!

History of Alternative Power Flow Solutions

- The power flow having alternative solutions is first mentioned in Power System Computational Conf. (PSCC) papers in 1972 and 1975, though there are earlier papers in Russian.
  - [a] V.I. Idelchik, V.I. Tarasov, “Experimental Investigations of Existence, Non-Uniqueness and Convergence of Solution of Power Flow Problem Equations in Power Systems”, 4th PSCC, Grenoble, FR, 1972.
  - [b] A. Klos, A. Kerner, “The Non-Uniqueness of Load-Flow Solution,” Proc. 5th PSCC, Cambridge, UK, 1975.

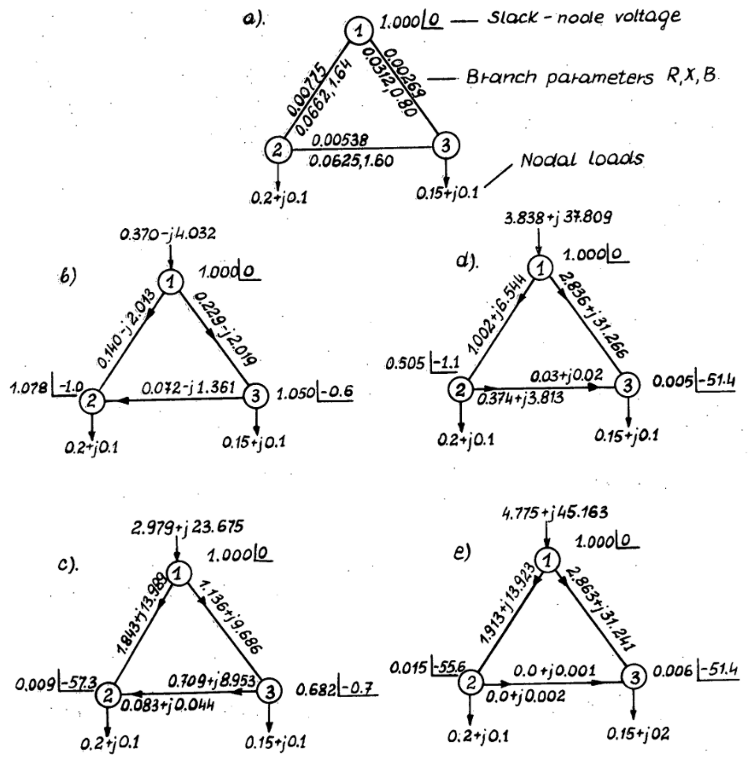


Fig 1 Example of the load-flow solutions in 3 - busbars network

a/ network data p.u. /base power 100 MVA; base voltage 400 kV/

b/,c/,d/,e/ load-flow solutions

Image source: [b]

Fractal NR Power Flow ROAs

- Starting in the late 1980’s, papers describing the fractal nature of the ROAs started to appear; my work was motivated by the book *Chaos: Making of a New Science* by James Gleick (1987).

Both images from [a]

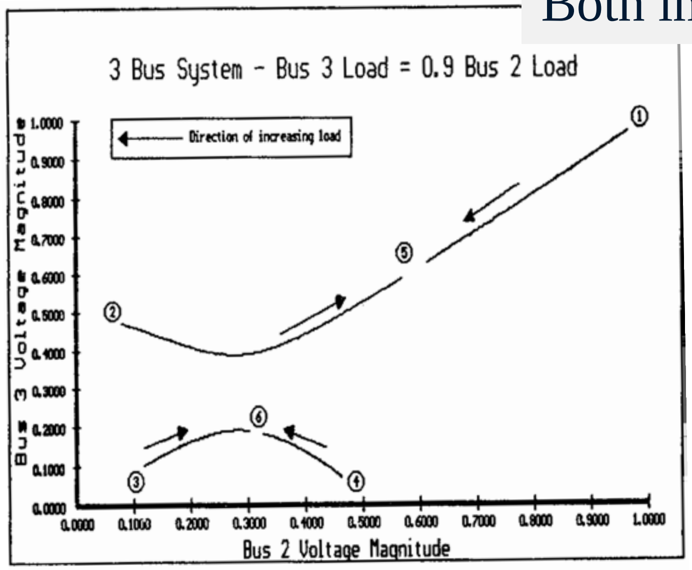


Figure 3

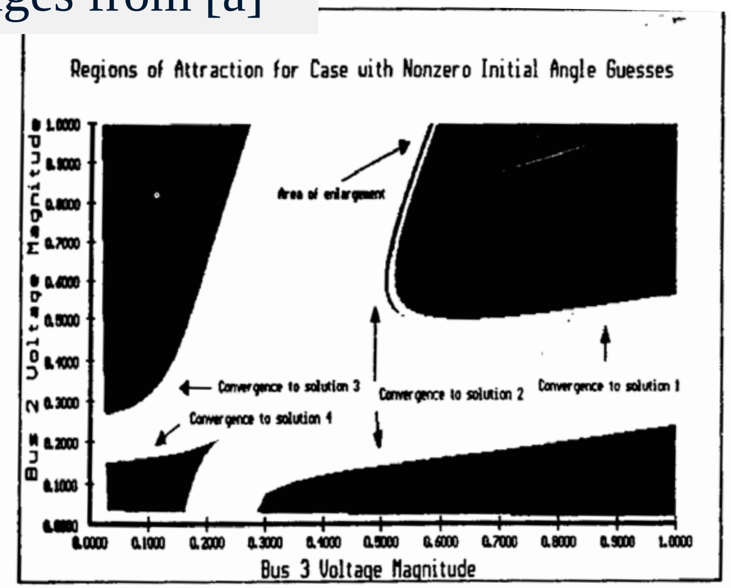


Figure 8

From [c]

Figure 5. 2π-by-2π grid of initial conditions in angle coded by the color of the load-flow solutions in Table 2

[a] C. L. DeMarco and T. J. Overbye, "Low Voltage Power flow Solutions and Their Role in Exit Time Based Security Measures for Voltage Collapse", Proc of the 27th Conference on Decision and Control (CDC), Austin, TX, Dec. 1988.

[b] J.S. Thorp, S.A. Naqavi, “Load Flow Fractals,” Proc. 28th Conference on Decision and Control (CDC), Tampa, FL, Dec. 1989.

[c] J.S. Thorp, S.A. Naqavi, “Load Flow Fractals Draw Clues to Erratic Behavior,” *IEEE Computer Applications Power*, Jan. 1997, pp. 59-62 (published in color)

How Many Alternative Solutions Exist?

- The short answer is “lots or “it depends on system loading.” In an unloaded system with n+1 buses, a separate AltS could exist for every combination of buses since zero power at each bus can be achieved by either zero current (open circuit) or zero voltage (short circuit); so potentially 2^n.
  - As the system loading increases, the AltSs coalesce, until eventually just a single AltS exists (along with the OprS)

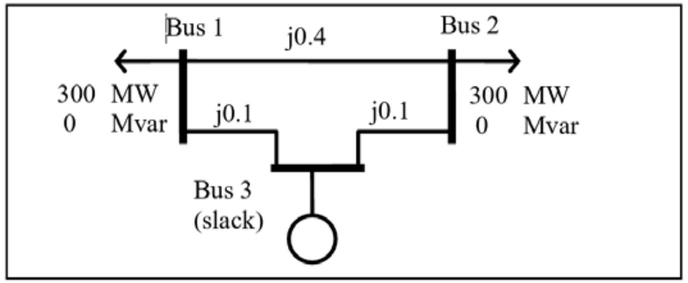


Figure 1: Three Bus System

Voltage	Solution A	Solution B	Solution C	Solution D
V_1	0.9487	0.2677	0.7477	0.3162
θ_1	-18.43°	-77.52°	-26.95°	-71.56°
V_2	0.9487	0.7471	0.2677	0.3162
θ_2	-18.43°	-26.95°	-77.52°	-71.56°

Table 1: Three Bus Base Case Solutions

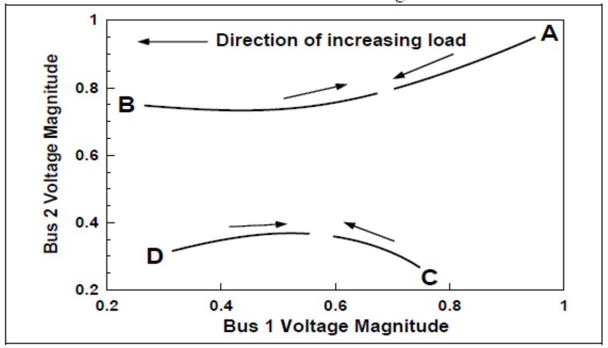


Figure 2: Maximum Load Participation at Bus 1

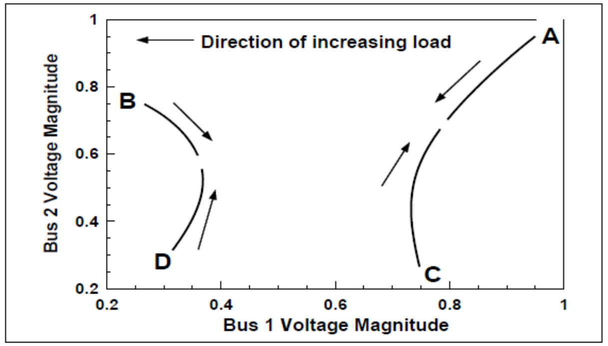


Figure 3: Maximum Load Participation at Bus 2

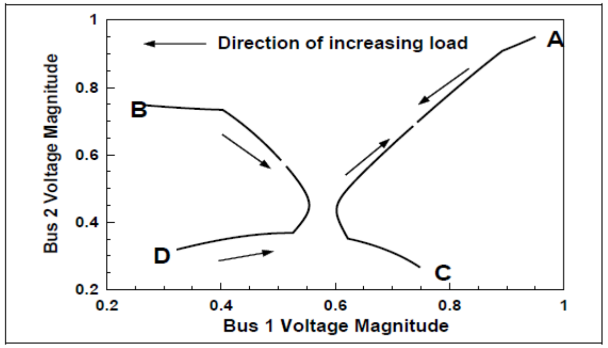


Figure 4: Varying Load Participation

Images from: T.J. Overbye and R.P. Klump, “Effective calculation of power system low voltage solutions,” IEEE Transactions on Power Systems, Vol. 11, pp. 75-82, February 1996.

Power Flow Solutions and Stability

- Power flow is a static analysis tool, so the concept of stability does not apply. However, if standard power system dynamics are assumed, the stability of a solution can be determined specified.
- Because with the standard dynamics the OprS is stable, the term stable equilibrium point (SEP) is often used synonymously with the OprS, and unstable equilibrium points (UEPs) with the AltSs.
- Using the approach of [a], the UEPs are assigned a type based on the number of positive eigenvalues in the Jacobian of the power flow equations; a type-one UEP would have a single positive eigenvalue.

[a] H. Chiang and J. S. Thorp, "The Closest Unstable Equilibrium Point Method for Power System Dynamic Security Assessment,“ IEEE Trans. Circuits and Systems, pp. 1187-1200, Vol. CAS-36, no. 9, Sep. 1989.

Application of AltSs

- In almost all situations the desire is to converge to the OprS; hence the goal of power flow algorithms and initial value selection are to ensure the NR starting point is within the OprS's ROA.
- Some exceptions existed, such as when determining power system voltage stability using energy methods. There the goal is to find to find at least a subset of the type-one AltSs.
  - An AltS, combined with the OprS, is used to determine an energy measure for the associated solution, giving a measure of the voltage stability in a portion of the system.

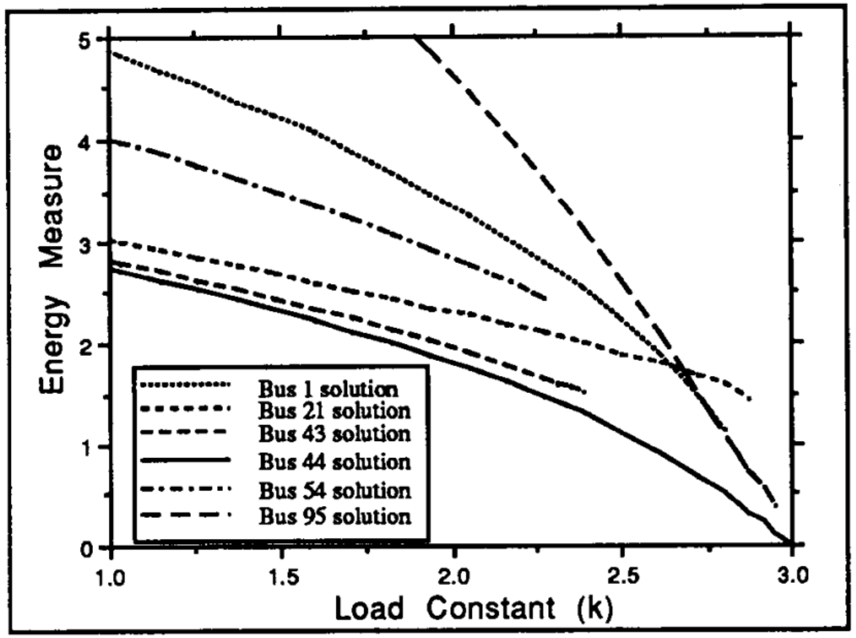


Figure 4-2

T. J. Overbye, C. L. DeMarco, "Improved techniques for power system voltage stability assessment using energy methods," in IEEE Transactions on Power Systems, vol. 6, no. 4, pp. 1446-1452, Nov. 1991

Bus Angle Variation

- In a large grid there can be substantial variation in the bus angles. This means that the flat start guess for a bus (1∠0°) may not be in the ROA if starting from a converged solution.
- As an example, the image on the right contours the angle variation across a synthetic 23,600 bus grid [a].
  - Here the reference bus is in Mississippi with an angle of -30 degrees.

Image source: K. Zhgun, S. Kunkolienkar, F. Safdarian, T.J. Overbye, J. Weber, “Improving Power Flow Convergence By Leveraging Region of Attraction Visualization,” 2025 North American Power Symposium (NAPS), Hartford, CT, October 2025. Case can be downloaded from electricgrids.engr.tamu.edu.

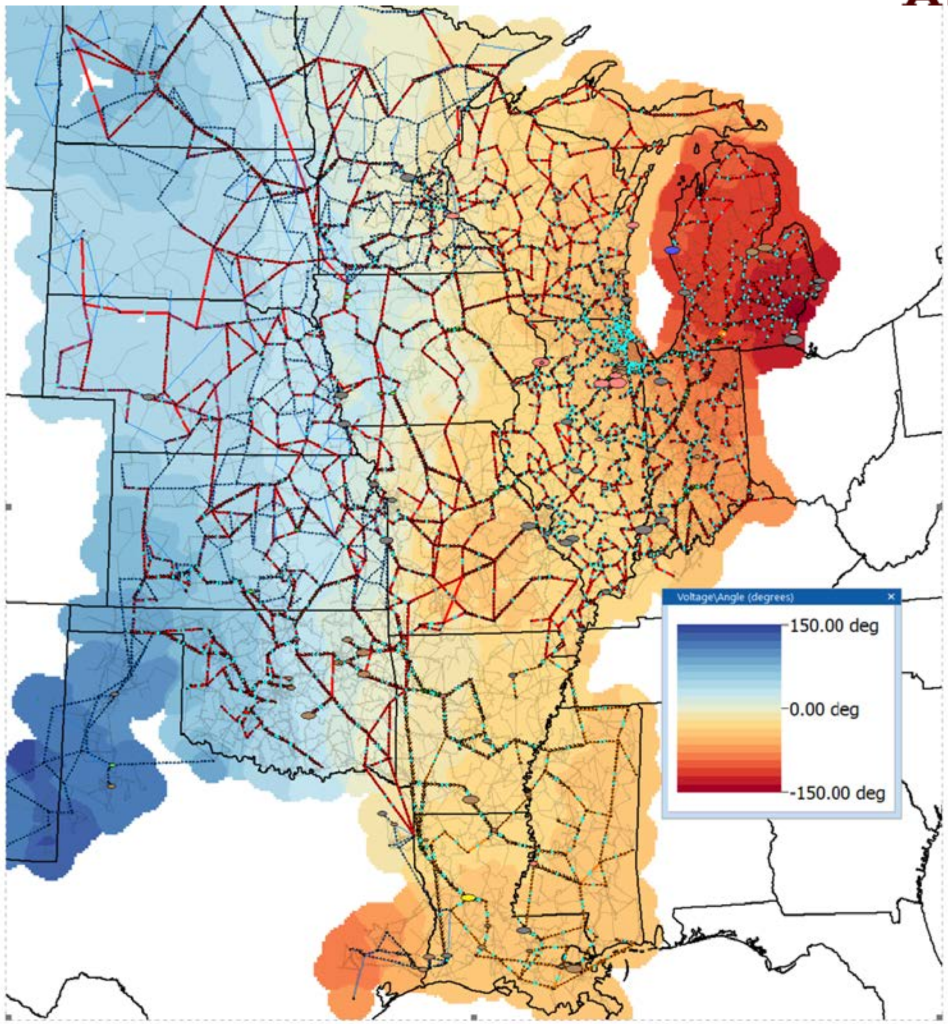
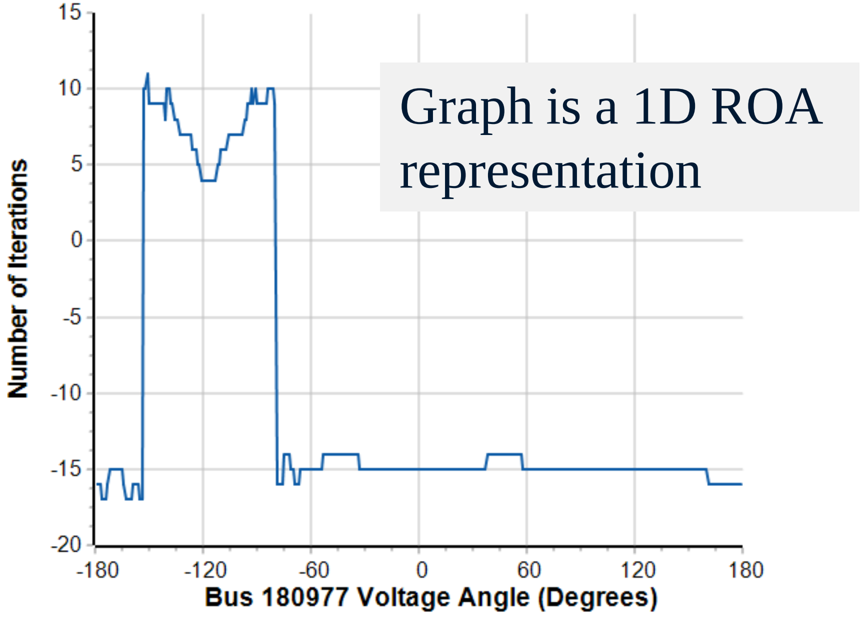
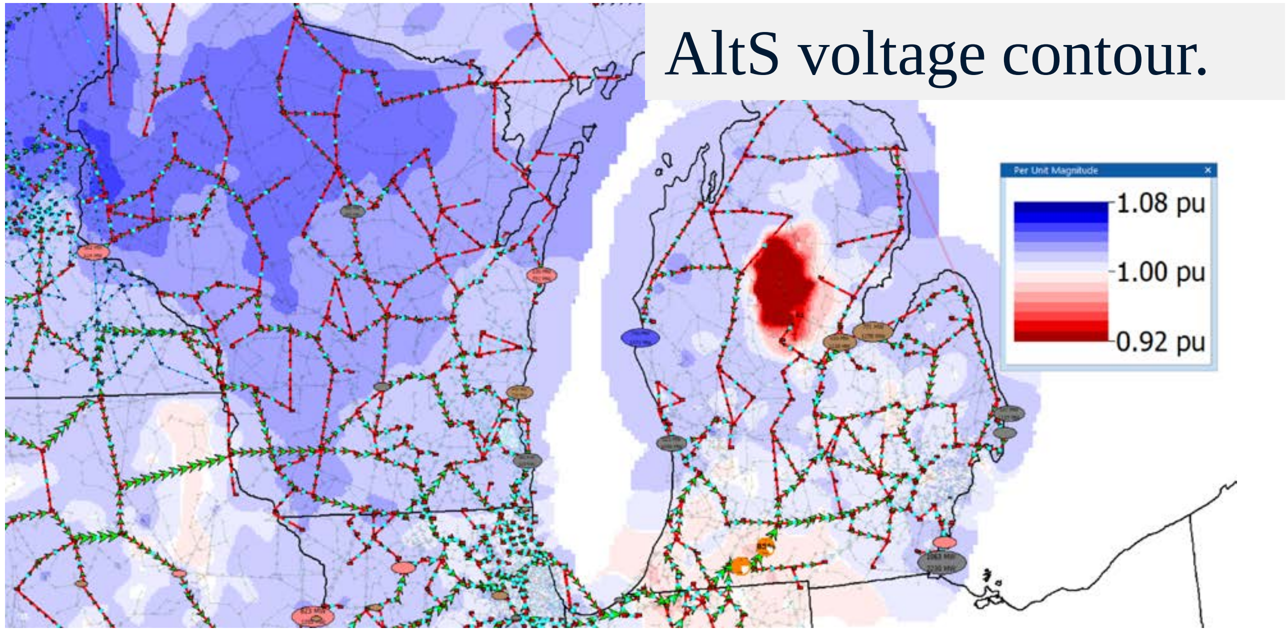


Figure 7: 23,600-Bus Electric Grid Voltage Angle Contour

Example: AltS for the 23,600 Bus Case

- With AltS prevention options in the power flow disabled, AltSs can be easily obtained for this case, with flat start cases at individual buses only converging to the OprS 38% of the time. The right graph shows the convergence behavior at one bus (180977 in Michigan with an OprS angle of -116.6°) (Negative values indicate AltS convergence).





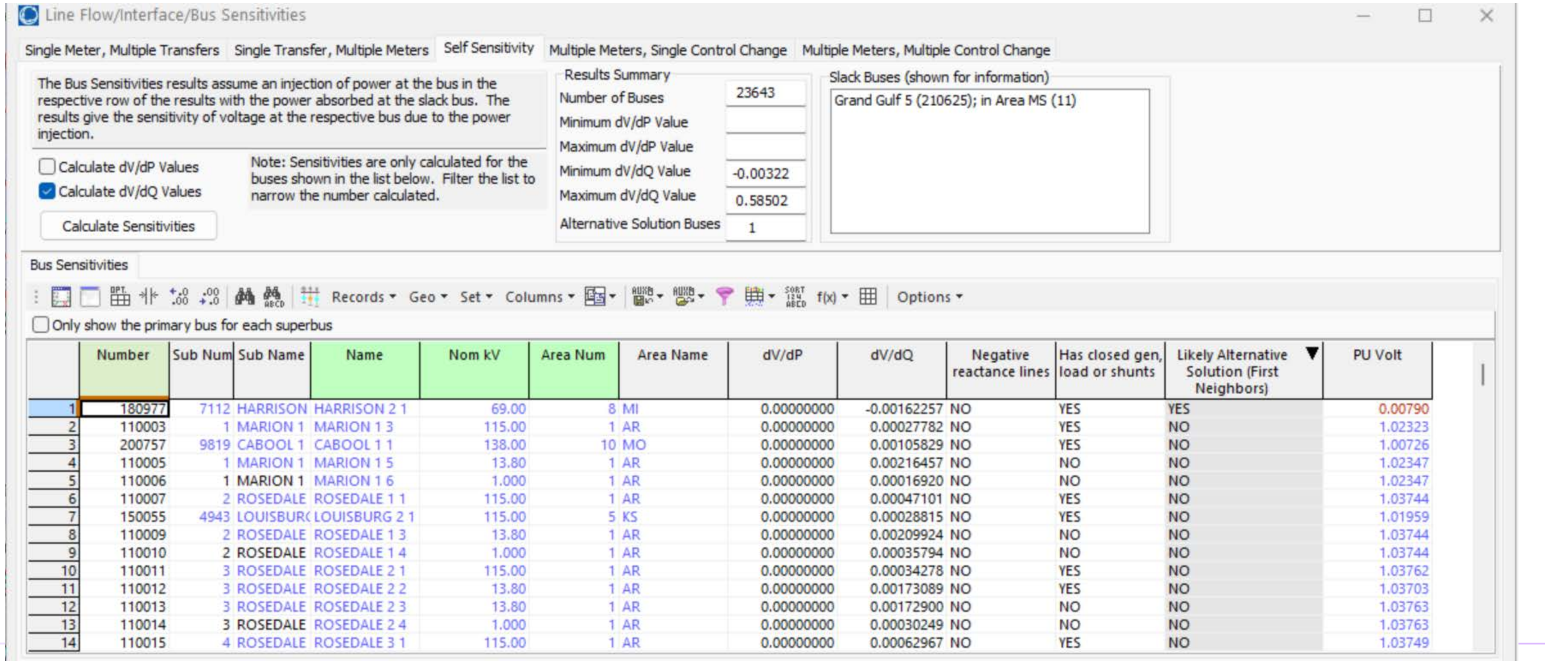
Detecting AltSs

- Since AltSs are often characterized by low voltages, sorting or contouring the bus voltages can provide a quick check; however, this can miss some.
- Fairly fast, more comprehensive algorithms exist for identifying AltSs, with a description of one algorithm given in [a]. The gist of the algorithm is to identify locations in the power system in which the dV_i/dQ_i values are negative (with Q_i assumed positive for power injected into the bus) except for locations that have negative reactance branches
  - Negative reactance branches occur with series capacitors and for some branches in three winding transformer equivalent circuits
  - The dV_i/dQ_i values can be calculated with order ln(n) computation using sparse vector methods; hence all buses can be checked in order n ln(n) (similar to a sort)

[a] K. Zhgun, F. Safdarian, J. Griffin, T.J. Overbye, “Identifying Problematic AC Power Flow Alternative Solutions in Large Power Systems”, 2023 North American Power Symposium, October 2023.

Detecting AltS in PowerWorld Simulator

- This algorithm is implemented in Simulator, and is available by selecting **Tools, Sensitivities, Flows and Voltage Sensitivities** to show the Sensitivities Dialog, selecting the Self Sensitivity page, and clicking **Calculate Sensitivities**.
  - Calculating the values for all 23,600 buses takes about 8 seconds



	Number	Sub Num	Sub Name	Name	Nom kV	Area Num	Area Name	dV/dP	dV/dQ	Negative reactance lines	Has closed gen, load or shunts	Likely Alternative Solution (First Neighbors)	PU Volt
1	180977	7112	HARRISON	HARRISON 2 1	69.00	8	MI	0.00000000	-0.00162257	NO	YES	YES	0.00790
2	110003	1	MARION 1	MARION 1 3	115.00	1	AR	0.00000000	0.00027782	NO	YES	NO	1.02323
3	200757	9819	CABOOL 1	CABOOL 1 1	138.00	10	MO	0.00000000	0.00105829	NO	YES	NO	1.00726
4	110005	1	MARION 1	MARION 1 5	13.80	1	AR	0.00000000	0.00216457	NO	NO	NO	1.02347
5	110006	1	MARION 1	MARION 1 6	1.000	1	AR	0.00000000	0.00016920	NO	NO	NO	1.02347
6	110007	2	ROSEDALE	ROSEDALE 1 1	115.00	1	AR	0.00000000	0.00047101	NO	YES	NO	1.03744
7	150055	4943	LOUISBUR(	LOUISBURG 2 1	115.00	5	KS	0.00000000	0.00028815	NO	YES	NO	1.01959
8	110009	2	ROSEDALE	ROSEDALE 1 3	13.80	1	AR	0.00000000	0.00209924	NO	NO	NO	1.03744
9	110010	2	ROSEDALE	ROSEDALE 1 4	1.000	1	AR	0.00000000	0.00035794	NO	NO	NO	1.03744
10	110011	3	ROSEDALE	ROSEDALE 2 1	115.00	1	AR	0.00000000	0.00034278	NO	YES	NO	1.03762
11	110012	3	ROSEDALE	ROSEDALE 2 2	13.80	1	AR	0.00000000	0.00173089	NO	YES	NO	1.03703
12	110013	3	ROSEDALE	ROSEDALE 2 3	13.80	1	AR	0.00000000	0.00172900	NO	NO	NO	1.03763
13	110014	3	ROSEDALE	ROSEDALE 2 4	1.000	1	AR	0.00000000	0.00030249	NO	NO	NO	1.03763
14	110015	4	ROSEDALE	ROSEDALE 3 1	115.00	1	AR	0.00000000	0.00062967	NO	YES	NO	1.03749

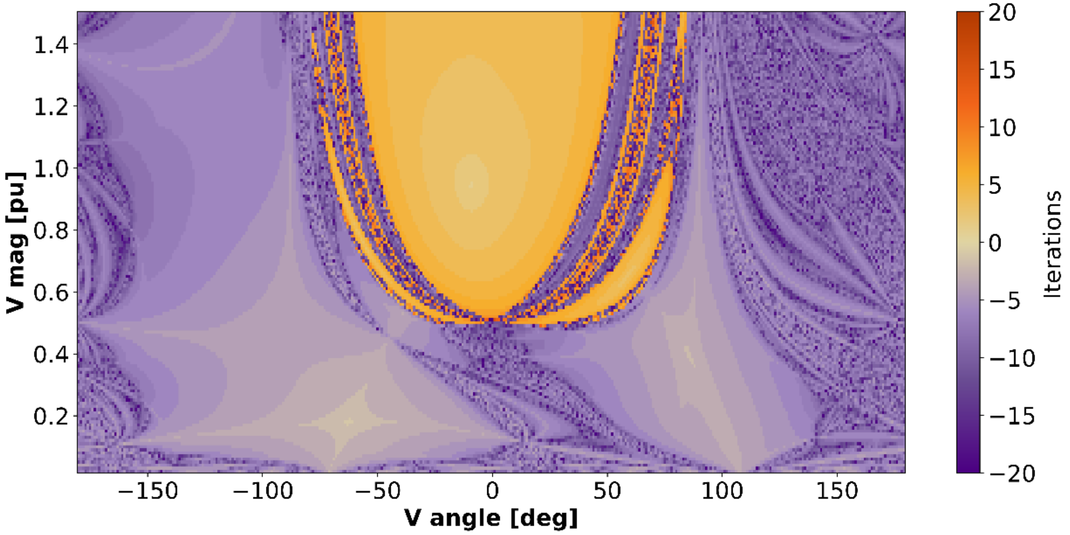
Avoiding AltSs

- As noted at the beginning, avoiding AltS involves a combination of
  - Utilizing available options to modify the power flow solution (Newton-Raphson here)
  - Avoiding, as much as possible, large power flow changes
  - When new buses are manually added, setting their initial angles close to the existing angle values
  - Effectively managing the explicitly modeled wye-delta phase shifts, denoted as phase shift groups
- Using the previously mentioned detection techniques can be helpful in keeping AltSs from persisting.
  - Once a power flow has solved to an AltS it is unlikely to resolve to the OprS without intervention

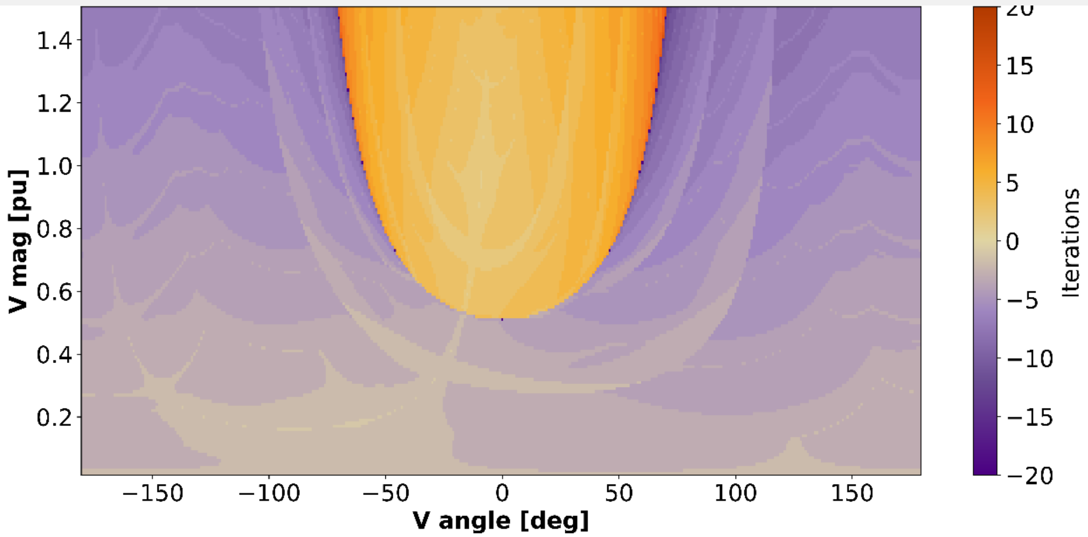
Impact of the NR Formulation on the Convergence

- As noted earlier the NR algorithm can be applied using either the polar or rectangular formulation of the power balance equations, and this does change the convergence behavior and the ROA; with the rectangular formulation the optimal multiplier can also be used [a].

Two-bus ROA with Polar NR



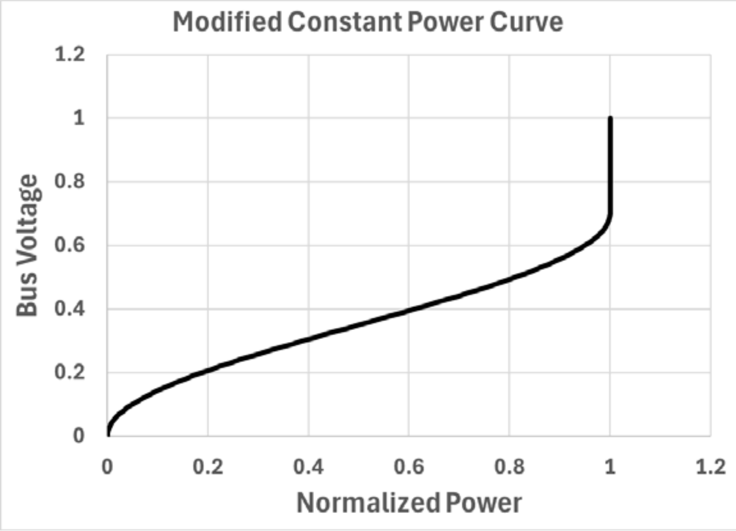
Two-bus ROA with Rectangular Polar NR with Opt. Mult.

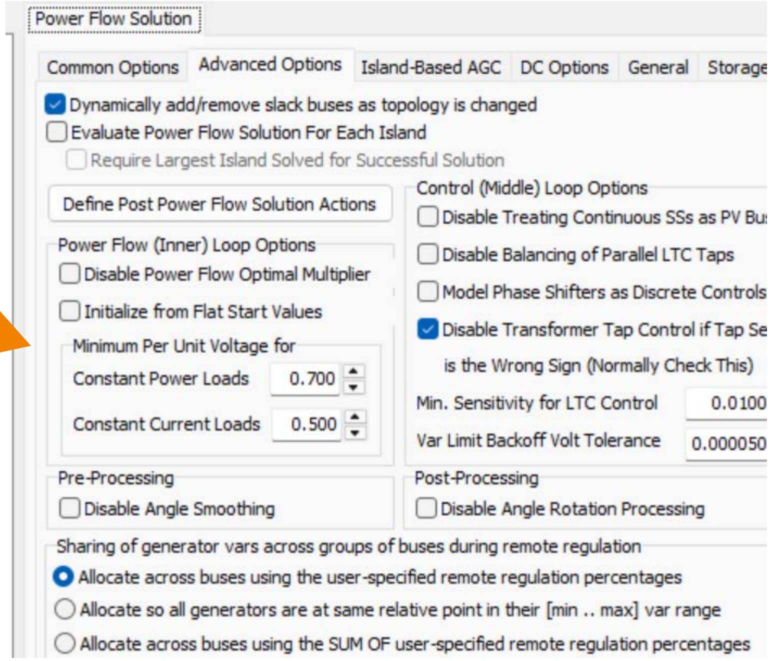


[a] S. Iwamoto and Y. Tamura, "A Load Flow Calculation Method for Ill-Conditioned Power Systems," *IEEE Trans. Power App. & Syst.* vol. PAS-100, no. 4, pp. 1736-1743, April 1981.

Some PowerWorld Simulator Solution Options

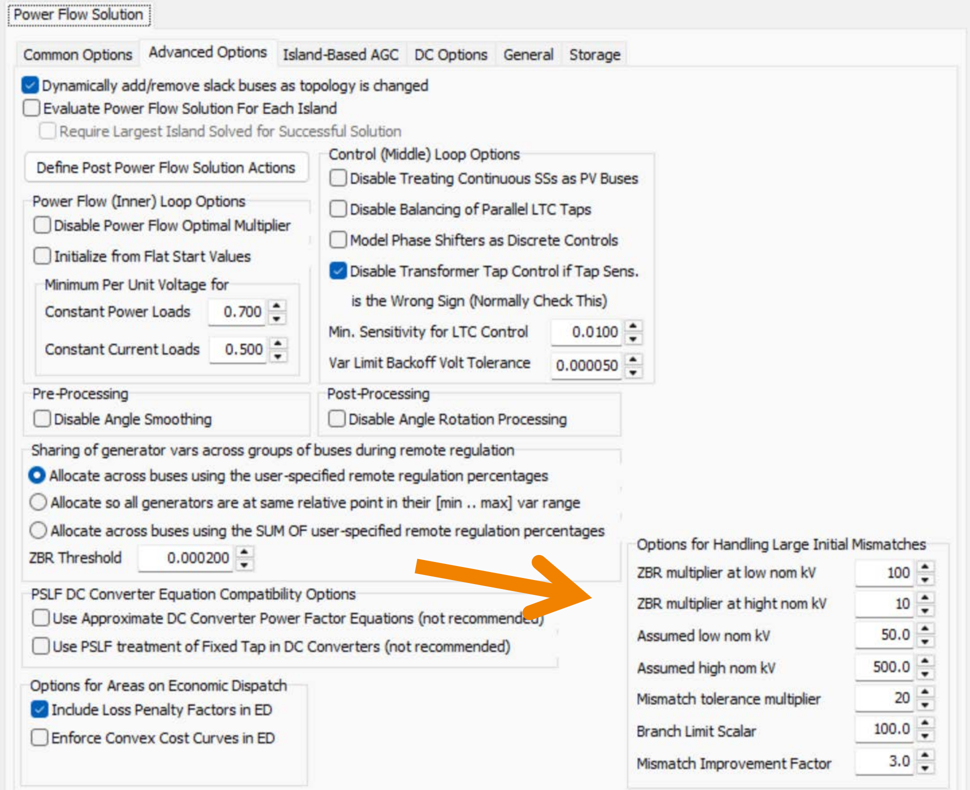
- Simulator has a number of options to prevent power flow divergence or AltS convergence, available at **Options, Solution Options**.
  - Usually the default values are best
- One commonly used option is to add voltage dependence to the constant power and constant current loads as their voltage drops; since the load goes to zero at zero volts, this means the short-circuit condition is an AltS.





Solution Options, cont.

- A newer option set is to do pre-processing on selected voltage guesses when there are large initial mismatches, likely due to some inconsistent initial voltage angle guesses.
- These options can be very useful in certain situations but are still being evaluated to make sure they "do no harm."
- The flat start procedure has recently been improved to better deal with the explicitly modeled angle shifts caused due to wye-delta transformers. This involves the code automatically calculating phase shift groups.



Phase Shift Groups

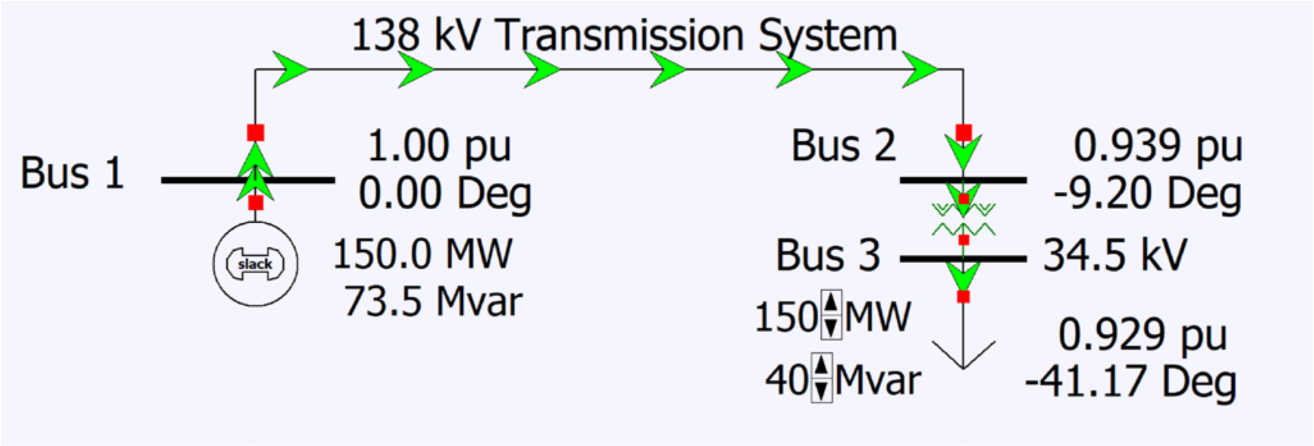
- In three-phase power systems it is very common to have two winding transformers that employ wye-connected windings on one side and delta-connected windings on the other, or to have three winding transformers with a combination of wye and delta windings.
  - Such connections introduce phases shifts with multiples of 30°
- In a standard (positive sequence) power flow these phase shifts to not need to be modeled; however, increasingly they are being modeled by explicitly adding multiple of 30° phase shifts to the transformers.
  - This results in corresponding phase shifts in the solution angles
- To help keep track of these phase shifts, PowerWorld has introduced a new data type known as Phase Shift Groups (PSGs).

Phase Shift Groups (PSGs), cont.

- A PSG is defined as the set of boundary transformers that are modeled as having a specific phase shift, and the lower voltage, radial subnetwork buses affected by this phase shift.
  - A simple, and quite common example, is single generator connected to the transmission system through a generator step-up transformer (GSU) that is grounded-wye on the high side, and delta on the low side
  - Another simple example is a radial load modeled with a step-down transformer
  - However, PSGs can contain larger subnetworks with many buses and several boundary transformers (e.g., a 34.5 kV network)
- A PSG can contain another PSG, having an even lower voltage.
- Transformers can only be part of the boundary for a single PSG, but buses can be in multiple PSGs.

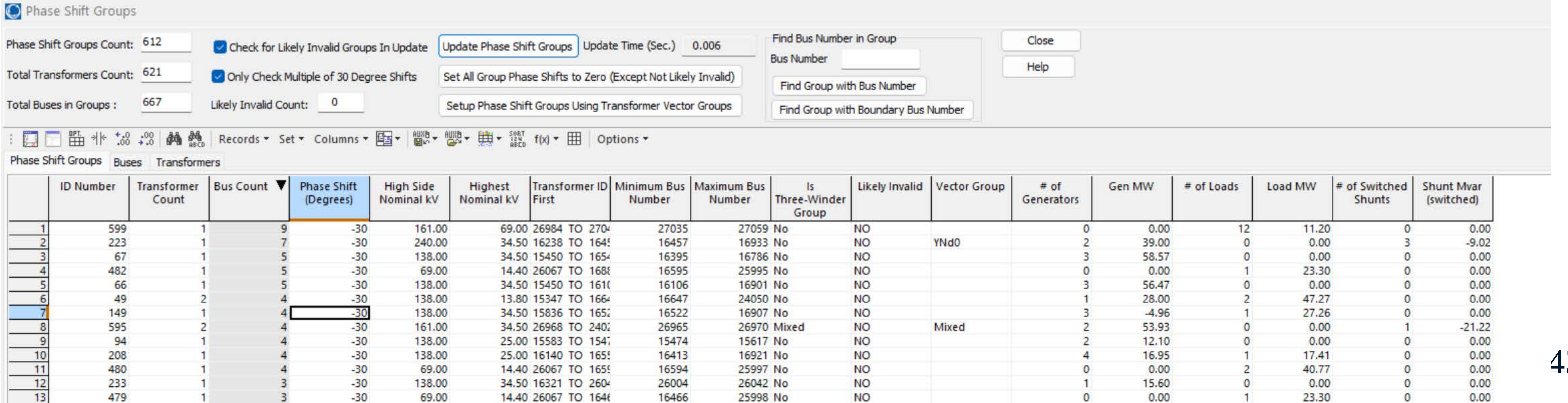
Simple PSG Example

- The below oneline shows an example in which a load is served through a 138/34.5 kV transformer (connected delta-grounded wye) in which the lower voltage lags the transmission system by 30 degrees.
  - This is modeled in the power flow by including a 30 degree phase shift with transformer going between Bus 2 and Bus 3.
  - This phase shift does not need to be modeled, and has no impact on the power flow solution other than shifting the Bus 3 voltage angle; however, commonly they are modeled
- Here the PSG has a single boundary trans-former and a single bus (i.e., Bus 3).



Phase Shift Groups Dialog

- PSGs are only calculated and shown in the **Run Mode**.
- To view all the PSGs in a case select **Tools, Connections, Find Phase Shift Groups** to display the Phase Shift Groups Dialog.
  - The below example is from a recent 27,000 bus WECC case with the bus numbers sequentially renumbered to obscure the actual system components.
  - The PSG calculation is quite fast, taking about 6 ms for the WECC.



	ID Number	Transformer Count	Bus Count	Phase Shift (Degrees)	High Side Nominal kV	Highest Nominal kV	Transformer ID First	Minimum Bus Number	Maximum Bus Number	Is Three-Winder Group	Likely Invalid	Vector Group	# of Generators	Gen MW	# of Loads	Load MW	# of Switched Shunts	Shunt Mvar (switched)
1	599	1	9	-30	161.00	69.00	26984 TO 2704	27035	27059	No	NO		0	0.00	12	11.20	0	0.00
2	223	1	7	-30	240.00	34.50	16238 TO 1645	16457	16933	No	NO	YNd0	2	39.00	0	0.00	3	-9.02
3	67	1	5	-30	138.00	34.50	15450 TO 1654	16395	16786	No	NO		3	58.57	0	0.00	0	0.00
4	482	1	5	-30	69.00	14.40	26067 TO 1688	16595	25995	No	NO		0	0.00	1	23.30	0	0.00
5	66	1	5	-30	138.00	34.50	15450 TO 1610	16106	16901	No	NO		3	56.47	0	0.00	0	0.00
6	49	2	4	-30	138.00	13.80	15347 TO 1664	16647	24050	No	NO		1	28.00	2	47.27	0	0.00
7	149	1	4	-30	138.00	34.50	15836 TO 1652	16522	16907	No	NO		3	-4.96	1	27.26	0	0.00
8	595	2	4	-30	161.00	34.50	26968 TO 2402	26965	26970	Mixed	NO	Mixed	2	53.93	0	0.00	1	-21.22
9	94	1	4	-30	138.00	25.00	15583 TO 1547	15474	15617	No	NO		2	12.10	0	0.00	0	0.00
10	208	1	4	-30	138.00	25.00	16140 TO 1655	16413	16921	No	NO		4	16.95	1	17.41	0	0.00
11	480	1	4	-30	69.00	14.40	26067 TO 1659	16594	25997	No	NO		0	0.00	2	40.77	0	0.00
12	233	1	3	-30	138.00	34.50	16321 TO 2604	26004	26042	No	NO		1	15.60	0	0.00	0	0.00
13	479	1	3	-30	69.00	14.40	26067 TO 1646	16466	25998	No	NO		0	0.00	1	23.30	0	0.00

PSG Applications

- Since these phase shifts do not affect the power flow solution, PSGs will likely initially be use for informational purposes.
  - Identifying the lower voltage radial subnetworks
- As transformer vector groups become more common, their phase shifts can now be easily added (or removed) from a case.
- Knowing the PSGs can helping in setting initial bus angles, avoiding AltSs.
- PSGs can help when comparing bus phase angles across a network.
- PSGs are needed when the power flow is used for initializing harmonic analysis (e.g., with the new GICHarm functionality).
- PSGs can also help with flat start solutions since the initial bus angles can be uniformly adjusted to account for these phase shifts.

Summary and Current Work

- By its nature the power flow usually has multiple solutions, with typically only the operable solution (OprS) desired.
- Avoiding the alterative solutions (AltSs) involves a combination of
  - Utilizing available options to modify the power flow solution (Newton-Raphson here)
  - Avoiding, as much as possible, large power flow changes; several smaller changes might be a better approach
  - When new buses are manually added setting their initial angles close to the existing angle value
  - Effectively managing the explicitly modeled wye-delta phase shifts
- Research is ongoing to improve the NR convergence behavior to increase the region of attraction for the OprS, and to restore an AltS to its OprS.

Thank You! Questions?

Various papers on this topic are available at **overbye.engr.tamu.edu/publications/**

For additional questions or comments email overbye@tamu.edu