Power System Economics and Market Modeling

M3: Overview of Linear Programming

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Motivation

- Simulator optimization tools are based on Linear Programming (LP) methods.
- There are other types of OPF algorithms, but LP is well suited for market applications.
- Reviewing some LP concepts will be useful to understand how the power system is optimized and how to interpret OPF results.

Linear Programming

- LP problems deal with maximizing or minimizing a linear objective function:

$$z = \overset{\text{Cost Vector}}{\overset{\downarrow}{\mathrm{c}}}^{\mathrm{T}}\overset{\text{LP Variables}}{\overset{\downarrow}{\mathrm{x}}}$$

- subject to a set of linear constraints:

$$\begin{array}{cccccc} a_{11}x_1 & +a_{12}x_2 & +\cdots & a_{1n}x_n & >,=,< & b_1 \\ \vdots & & \ddots & & \vdots & \vdots \\ \multicolumn{4}{c}{\underbrace{a_m x_1 \quad +a_{m2}x_2 \quad +\cdots \quad a_{mn}x_n}} & >,=,< & b_m \end{array}$$

$$\mathbf{Ax} \;\; >,=,< \;\; \mathbf{b}$$

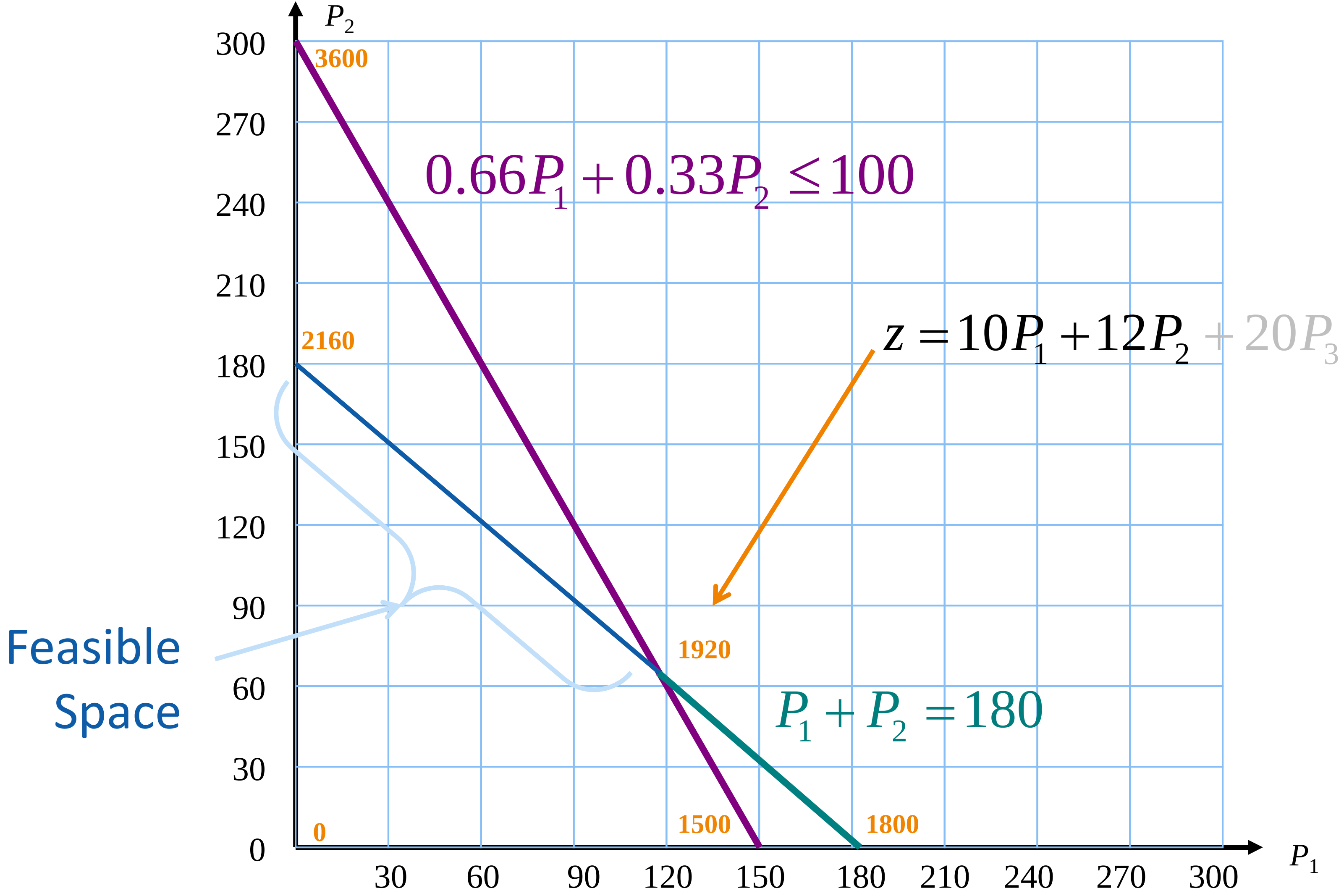
Example: B3LP

- Three generator controls P_1, P_2, P_3
- Incremental costs of 10, 12, 20 \$/MWh, respectively

$$
\begin{array}{lrll}
\text{min:} & 10P_1 + 12P_2 + 20P_3 & & \\
\text{st:} & P_1 + P_2 + P_3 & = 180 & \text{Power Balance} \\
 & 0.66P_1 + 0.33P_2 & \leq 100 & \text{Line 1-3 Constraint} \\
 & P_1, P_2, P_3 & \geq 0 &
\end{array}
$$

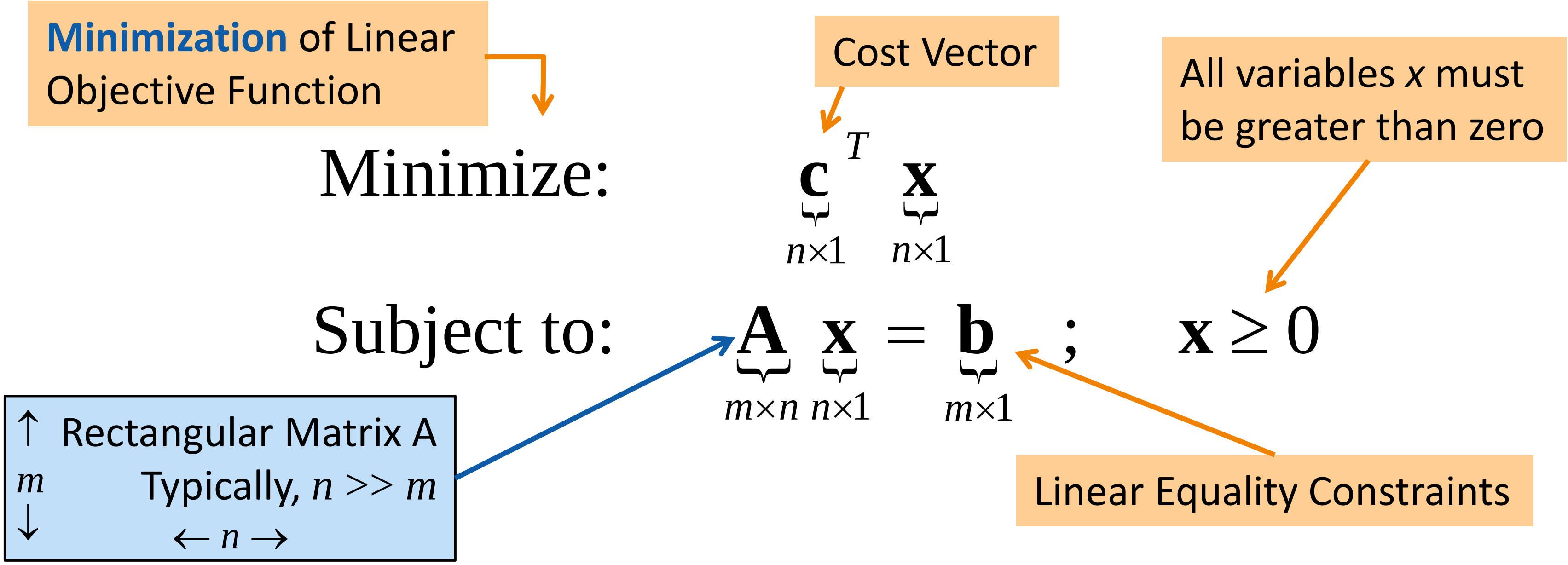
- Let us explore this problem graphically.

Example: B3LP



LP Standard Form

- The algorithm used to solve LP problems requires that the problem be stated in ***Standard Form***.



Minimization of Linear Objective Function

Cost Vector

All variables *x* must be greater than zero

Minimize: $\underbrace{\mathbf{c}}_{n\times 1}{}^{T} \; \underbrace{\mathbf{x}}_{n\times 1}$

Subject to: $\underbrace{\mathbf{A}}_{m\times n} \underbrace{\mathbf{x}}_{n\times 1} = \underbrace{\mathbf{b}}_{m\times 1} \; ; \quad \mathbf{x} \geq 0$

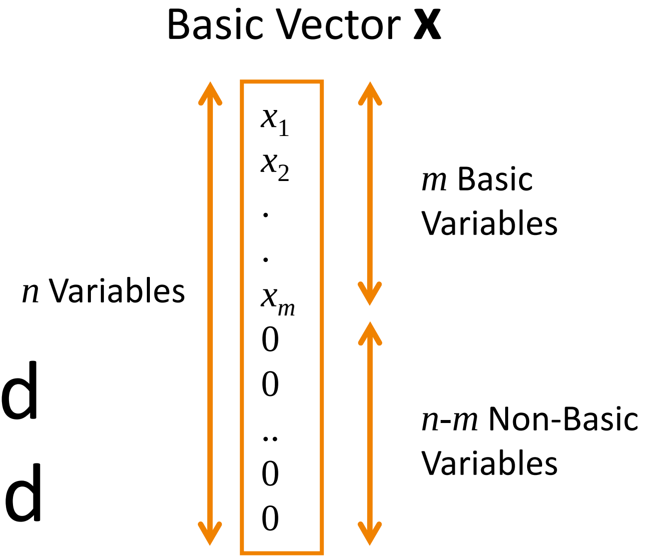
↑ Rectangular Matrix A
m Typically, $n >> m$
↓ $\leftarrow n \rightarrow$

Linear Equality Constraints

- To describe the LP solution algorithm, we need a few definitions.

LP Standard Form

- A vector **x** is basic if **Ax = b** and at most ***m*** components of **x** are nonzero.
- We can order **x** so the basic and non-basic variables are grouped together.



Basic Vector **x**

x_1
x_2
.
.
x_m
0
0
..
0
0

n Variables

m Basic Variables

n-*m* Non-Basic Variables

$$\mathbf{x} = \begin{bmatrix}\mathbf{x}_B \\ \mathbf{x}_N\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\mathbf{A}_B & \mathbf{A}_N\end{bmatrix}\begin{bmatrix}\mathbf{x}_B \\ \mathbf{x}_N\end{bmatrix} = \mathbf{b}$$

Since $\mathbf{x}_N = 0, \quad \mathbf{x}_B = \mathbf{A}_B^{-1}\mathbf{b}$ $\quad$ ($\mathbf{A}_B$ must be nonsingular)

LP Standard Form

- Example: Find all the basic solutions of:

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} : \quad \text{Assume that one variable is zero.}$$

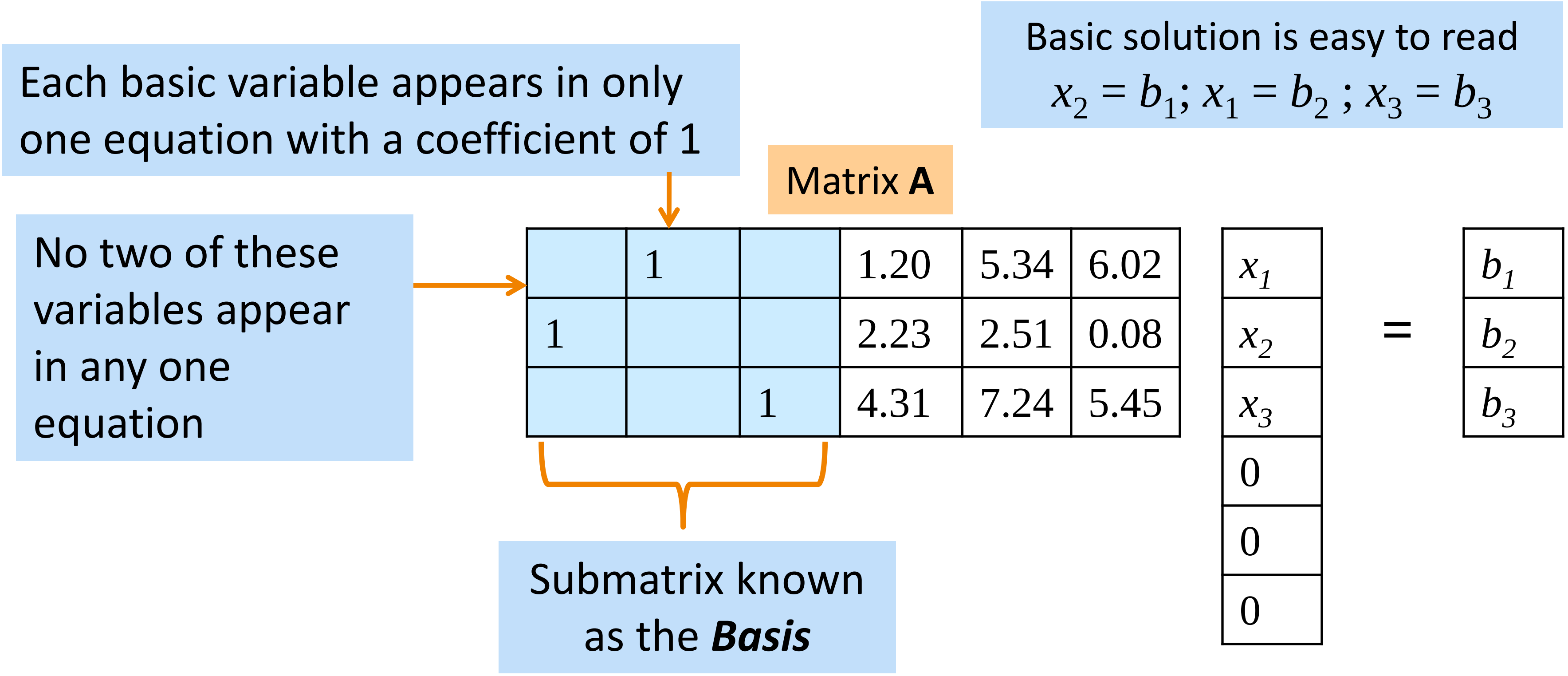
$$1.\ x_3 = 0,\ \mathbf{A}_3 = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix};\ \mathbf{A}_3^{-1} = \frac{1}{7}\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2.29 \\ 0.86 \\ \mathbf{0.00} \end{bmatrix}$$

$$2.\ x_2 = 0,\ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2.8 \\ \mathbf{0.0} \\ 1.2 \end{bmatrix}; \qquad 3.\ x_1 = 0,\ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \mathbf{0.00} \\ 4.67 \\ 5.33 \end{bmatrix}$$

- A vector **x** is feasible if **Ax = b** and **x** $\geq$ **0**.
- A basic solution is not necessarily feasible and a feasible solution is not necessarily basic.

LP Standard Form

- Canonical Form



Basic solution is easy to read

$x_2 = b_1; x_1 = b_2 \,; x_3 = b_3$

Each basic variable appears in only one equation with a coefficient of 1

Matrix **A**

No two of these variables appear in any one equation

$$\begin{bmatrix} & 1 & & 1.20 & 5.34 & 6.02 \\ 1 & & & 2.23 & 2.51 & 0.08 \\ & & 1 & 4.31 & 7.24 & 5.45 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Submatrix known as the ***Basis***

LP Standard Form

- The LP Standard form requires equality constraints
- ***Slack variables*** are used to convert an inequality to an equality.

Example 1:

$x_i \le b_j , \quad x_i \ge 0$ can be written as:

$x_i + x_s = b_j , \quad x_i , \; x_s \ge 0;$ where x_s is called the slack variable.

Example 2:

$50 \le \; x_i \le 100, \quad x_i \ge 0$ can be written as:

$x_i + x_{s1} = 100$

$x_i - x_{s2} = 50, \quad x_i , \; x_{s1}, x_{s2} \ge 0$

LP Standard Form

Example 3: B3LP

Rewrite in standard form:

Minimize: $$10P_1 + 12P_2 + 20P_3$$

Subject to:
$$\begin{aligned} P_1 + P_2 + P_3 \quad &= 180 \\ 0.66P_1 + 0.33P_2 + s &= 100 \\ \mathbf{x}, s \quad &\geq 0 \end{aligned}$$

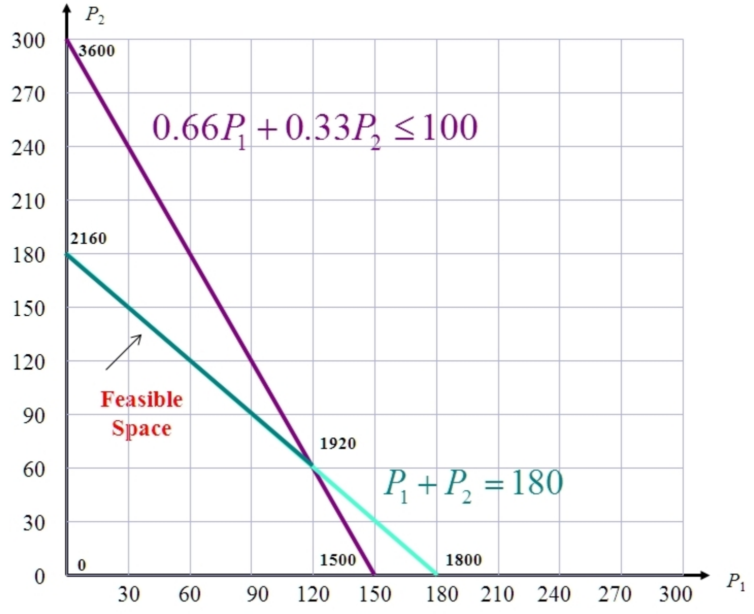
$$\mathbf{A} = \begin{bmatrix} P_1 & P_2 & P_3 & s \\ 1 & 1 & 1 & 0 \\ 0.66 & 0.33 & 0 & 1 \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 180 \\ 100 \end{bmatrix}; \quad \mathbf{c} = \begin{bmatrix} 10 \\ 12 \\ 20 \\ 0 \end{bmatrix}$$

Fundamental Theorem of LP

- Given a linear program in standard form with **A** $m \times n$ of rank m:
  - If there is a feasible solution, then there is a basic feasible solution
  - If there is an optimal feasible solution, then there is an optimal basic feasible solution.
- In other words, to find the optimal solution, we only need to look at the set of basic feasible solutions.

The Simplex Method

- Invented in 1940s by George Dantzig.
- The main idea is to move from one basic feasible solution to another with lower cost.
- Basic feasible solutions are at the vertices of a polytope.
- At least one of the feasible solutions is optimal.



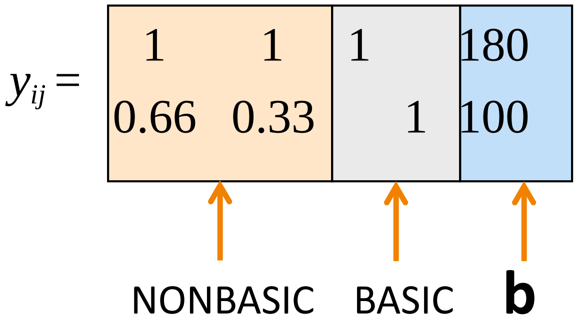
The Simplex Method

- The LP ***Tableau*** is used to move from one basic feasible solution to another.
- Append to matrix **A** the **b** vector as the last column.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & \\ 0.66 & 0.33 & & 1 \end{bmatrix};\ \mathbf{b} = \begin{bmatrix} 180 \\ 100 \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} 10 \\ 12 \\ 20 \\ 0 \end{bmatrix}$$

Tableau:



$$y_{ij} = \left[\begin{array}{cc|cc|c} 1 & 1 & 1 & & 180 \\ 0.66 & 0.33 & & 1 & 100 \end{array}\right]$$

The Simplex Method

- ***Pivoting*** is used to move from one basic solution to another, by changing the set of basic variables. It is based upon algebraic row operations.
- Suppose we would like to replace x_p in the basis with x_q (make x_q basic and x_p nonbasic).
- Pivoting is done in three steps:
  - Identify a pivot element y_{pq}
  - Normalize the row of the pivot to make $y_{pq} = 1$
  - Perform row operations to zero out all other elements of the pivot (column q).

The Simplex Method

Pivoting Example

$$y_{ij} = \left[\begin{array}{cc|cc|c} 1 & 1 & 1 & 0 & 180 \\ 0.66 & 0.33 & 0 & 1 & 100 \end{array}\right]$$

Currently P_3, and s are basic variables, while P_1 and P_2 are nonbasic. Suppose we want to replace P_3 by P_2. Pivot is in column of P_2 $(q = 2)$ and row corresponding to P_3 $(p = 1)$.

1. Identify y_{12} as pivot
$$\begin{bmatrix} 1 & \boxed{1} & 1 & 0 & 180 \\ 0.66 & 0.33 & 0 & 1 & 100 \end{bmatrix}$$

2. Normalize row 1: Already Normalized

3. Zero column 2
$$\begin{bmatrix} 1 & \boxed{1} & 1 & 0 & 180 \\ 0.33 & 0 & -0.33 & 1 & 40 \end{bmatrix}$$

The Simplex Method

Select the Exiting Basic Variable

- Assume we know that x_q must enter the basis. We need to determine the basic variable that should exit.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_m, 0, \ldots 0)$ be a basic feasible solution:

$$\mathbf{Ax} = \mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \cdots + \mathbf{a}_m x_m = \mathbf{b}$$

x_q will multiply its column $\mathbf{a}_q$, which can be written as:

$$\mathbf{a}_q = y_{1q}\mathbf{a}_1 + y_{2q}\mathbf{a}_2 + \ldots + y_{mq}\mathbf{a}_m$$

Multiplying this by ε>0 and substract:

$$(x_1 - \varepsilon y_{1q})\mathbf{a}_1 + (x_2 - \varepsilon y_{2q})\mathbf{a}_2 + \ldots + (x_m - \varepsilon y_{mq})\mathbf{a}_m + \varepsilon \mathbf{a}_q = \mathbf{b}$$

The Simplex Method

$$(x_1 - \varepsilon y_{1q})\mathbf{a}_1 + (x_2 - \varepsilon y_{2q})\mathbf{a}_2 + \ldots + (x_m - \varepsilon y_{mq})\mathbf{a}_m + \varepsilon \mathbf{a}_q = \mathbf{b}$$

As ε increases, the coefficientes change. If $y_{iq} > 0,$ then for some value of ε

$$\varepsilon = \min \left\{ \frac{x_i}{y_{iq}} : \; y_{iq} > 0 \right\}$$

the coefficient of column i is zero, hence x_i should leave the basis

The values of $x_i - \varepsilon y_{iq}$ are the new values of the basic variables

The Simplex Method

Select the Entering Nonbasic Variable

To determine the entering nonbasic variable we want to see how the cost function varies if we change their values.

$$\text{max: } z = \mathbf{c}_B{}^T \mathbf{x}_B + \mathbf{c}_N{}^T \mathbf{x}_N \qquad \text{s.t: } \begin{bmatrix} \mathbf{A}_B & \mathbf{A}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{bmatrix} = \mathbf{b}$$

$$\mathbf{A}_B \mathbf{x}_B + \mathbf{A}_N \mathbf{x}_N = \mathbf{b} \quad ; \quad \mathbf{A}_B \mathbf{x}_B = \mathbf{b} - \mathbf{A}_N \mathbf{x}_N$$

$$\rightarrow \quad \mathbf{x}_B = \mathbf{A}_B{}^{-1} \left(\mathbf{b} - \mathbf{A}_N \mathbf{x}_N \right)$$

$$z = \mathbf{c}_B{}^T \mathbf{A}_B{}^{-1} \left(\mathbf{b} - \mathbf{A}_N \mathbf{x}_N \right) + \mathbf{c}_N{}^T \mathbf{x}_N$$

$$z = \mathbf{c}_B{}^T \mathbf{A}_B{}^{-1} \mathbf{b} + \left(\mathbf{c}_N{}^T - \mathbf{c}_B{}^T \mathbf{A}_B{}^{-1} \mathbf{A}_N \right) \mathbf{x}_N$$

$$z = \mathbf{c}_B{}^T \mathbf{A}_B{}^{-1} \mathbf{b} + \mathbf{r}_N{}^T \mathbf{x}_N$$

REDUCED COST

Change in objective function due to $\mathbf{x}_N$

The Simplex Method

In the initial tableau in canonical form, $\mathbf{A}_B$ is the identity matrix of size $m \times m$. Thus $\mathbf{A}_B^{-1}$ is again the identity matrix. Consequently: $\boxed{\mathbf{r}_N^T = \mathbf{c}_N^T - \mathbf{c}_B^T \mathbf{A}_N}$

$\rightarrow$ Chose the variable with the most negative reduced cost to enter the basis. If all the reduced costs are ≥ 0 then we are at the optimal solution.

- Relative costs need to be updated after each pivot, which can be done by simply adding an additional row to the tableau, consisting of the reduced costs for the nonbasic variables and zeros for the basic variables.
- Finally, we add the negative of the cost function in the bottom right-hand corner of the tableau.

The Simplex Method

BASIC

NONBASIC

x_1	x_2	x_3	x_4	x_5	x_6	x_7	$\mathbf{b}$
1	0	0	a_{14}	a_{15}	a_{16}	a_{17}	b_1
0	1	0	a_{24}	A_{25}	a_{26}	a_{27}	b_2
0	0	1	a_{34}	A_{35}	a_{36}	a_{37}	b_3
0	0	0	r_4	R_5	r_6	r_7	$-\mathbf{c}^T\mathbf{x}$

Reduced
Costs

Negative of
Objective Function

Simplex Algorithm

- Start with a basic feasible solution
- Set up the initial tableau
  - Setup **A** and **b**
  - Calculate the initial reduced cost for the nonbasic variables
  - Set bottom right-hand element to negative of initial cost
- While at least one $r_i < 0$ do
  - Find variable to enter basis as the one with the most negative cost (q)
  - Use ε-test to determine the variable that should leave the basis (p)
  - Pivot on element y_{pq}

B3LP Unconstrained

- Three generator controls P_1, P_2, P_3
- Incremental costs of 10, 12, 20 \$/MWh, respectively

$$
\begin{aligned}
\text{min:} \quad & 10P_1 + 12P_2 + 20P_3 & \\
\text{st:} \quad & P_1 + P_2 + P_3 & = 180 \\
& P_1, P_2, P_3 & \geq 0
\end{aligned}
$$

- Select P_1 as the basic variable.
- Basic feasible solution P_1 = 180

Reduced Costs > 0
Solution is optimal

$$
\mathbf{A} = \begin{bmatrix} \underbrace{1}_{\mathbf{A}_B} & \underbrace{1 \quad 1}_{\mathbf{A}_N} \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 180 \end{bmatrix} \; ; \quad \mathbf{c} = \begin{bmatrix} 10 \} \mathbf{c}_B \\ \left. \begin{matrix} 12 \\ 20 \end{matrix} \right\} \mathbf{c}_N \end{bmatrix}
$$

$$
\mathbf{r}_N{}^T = \mathbf{c}_N{}^T - \mathbf{c}_B{}^T \mathbf{A}_N = \begin{bmatrix} 12 & 20 \end{bmatrix} - \begin{bmatrix} 10 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 10 \end{bmatrix}
$$

B3LP with Line Constraint

- Introduce the transmission line constraint:

$$\begin{array}{lll} \text{min:} & 10P_1 + 12P_2 + 20P_3 & \\ \text{st:} & P_1 + P_2 + P_3 & = 180 \\ & 0.66P_1 + 0.33P_2 + s & = 100 \\ & P_1, P_2, P_3, s_1 & \geq 0 \end{array}$$

$$\mathbf{A} = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 1 \\ 0.66 & 0.33 \end{matrix}}_{\mathbf{A}_N} & \underbrace{\begin{matrix} 1 & \\ & 1 \end{matrix}}_{\mathbf{A}_B} \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 180 \\ 100 \end{bmatrix} \quad ; \quad \mathbf{c} = \begin{bmatrix} \left.\begin{matrix} 10 \\ 12 \end{matrix}\right\} \mathbf{c}_N \\ \left.\begin{matrix} 20 \\ 0 \end{matrix}\right\} \mathbf{c}_B \end{bmatrix}$$

B3LP Revisited

We first calculate the reduced costs:

$$\mathbf{r}_N{}^T = \mathbf{c}_N{}^T - \mathbf{c}_B{}^T \mathbf{A}_N = \begin{bmatrix} 10 & 12 \end{bmatrix} - \begin{bmatrix} 20 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0.66 & 0.33 \end{bmatrix} = \begin{bmatrix} -10 & -8 \end{bmatrix}$$

The initial tableau is then: $\mathbf{y} =$

$$\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 180 \\ \boxed{0.66} & 0.33 & 0 & 1 & \boxed{100} \\ \boxed{-10} & -8 & 0 & 0 & -3600 \end{array} \rightarrow -\mathbf{c}_B{}^T \mathbf{x}_B$$

1. Pick Column with -10

2. Pick Row with 100/.66

To solve using the simplex algorithm, first pick the variable with most negative r to enter the basis ($q = 1$). Use the ε-test with column q to determine the exiting variable p:

$$\varepsilon = \min\left\{\frac{x_i}{y_{iq}} : y_{iq} > 0\right\} = \min\left\{\frac{180}{1}, \frac{100}{0.66}\right\} \quad \rightarrow \quad p = 2$$

B3LP Revisited

Pivot on element y_{21}:

$$\left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 180 \\ \boxed{0.66} & 0.33 & 0 & 1 & 100 \\ -10 & -8 & 0 & 0 & -3600 \end{array}\right] :$$

Normalize Row 2

$$\left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 180 \\ \boxed{1} & 0.5 & 0 & 1.5 & 150 \\ -10 & -8 & 0 & 0 & -3600 \end{array}\right] :$$

Zero Column 1

$$\left[\begin{array}{cc|ccc} 0 & 0.5 & 1 & -1.5 & 30 \\ \boxed{1} & 0.5 & 0 & 1.5 & 150 \\ 0 & -3 & 0 & 15 & -2100 \end{array}\right] :$$

Current basic variables are

$P_1 = 150$ and $P_3 = 30$

B3LP Revisited

Most negative reduced cost in column 2, Hence $q = 2$

$$\varepsilon = \min\left\{\frac{30}{0.5}, \frac{150}{0.5}\right\} \to \quad p = 1: \text{ Pivot on } y_{12}$$

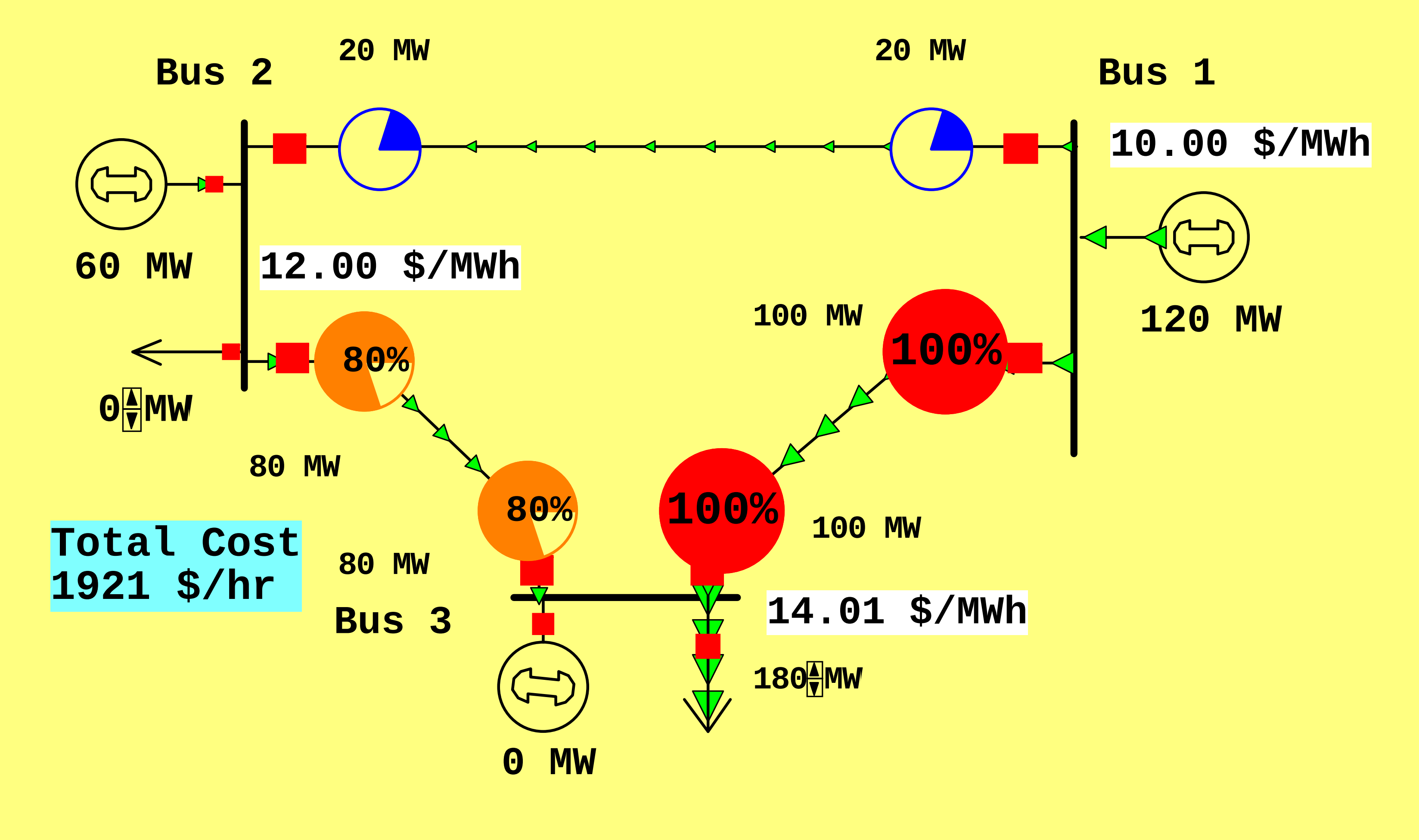
$$\left[\begin{array}{cc|ccc} 0 & \boxed{0.5} & 1 & -1.5 & 30 \\ 1 & 0.5 & 0 & 1.5 & 150 \\ 0 & -3 & 0 & 15 & -2100 \end{array}\right] : \quad \text{Normalize Row 1}$$

$$\left[\begin{array}{cc|ccc} 0 & \boxed{1} & 2 & -3 & 60 \\ 1 & 0.5 & 0 & 1.5 & 150 \\ 0 & -3 & 0 & 15 & -2100 \end{array}\right] : \quad \text{Zero Column 2}$$

$$\left[\begin{array}{cc|ccc} 0 & \boxed{1} & 2 & -3 & 60 \\ 1 & 0 & -1 & 3 & 120 \\ 0 & 0 & 6 & 6 & -1920 \end{array}\right] : \quad \begin{array}{c}\text{Basic Variables are} \\ P_1 = 120, P_2 = 60\end{array}$$

Optimal Solution is
P_1=120, P_2=60, P_3=0
Total Cost = $1920/hr

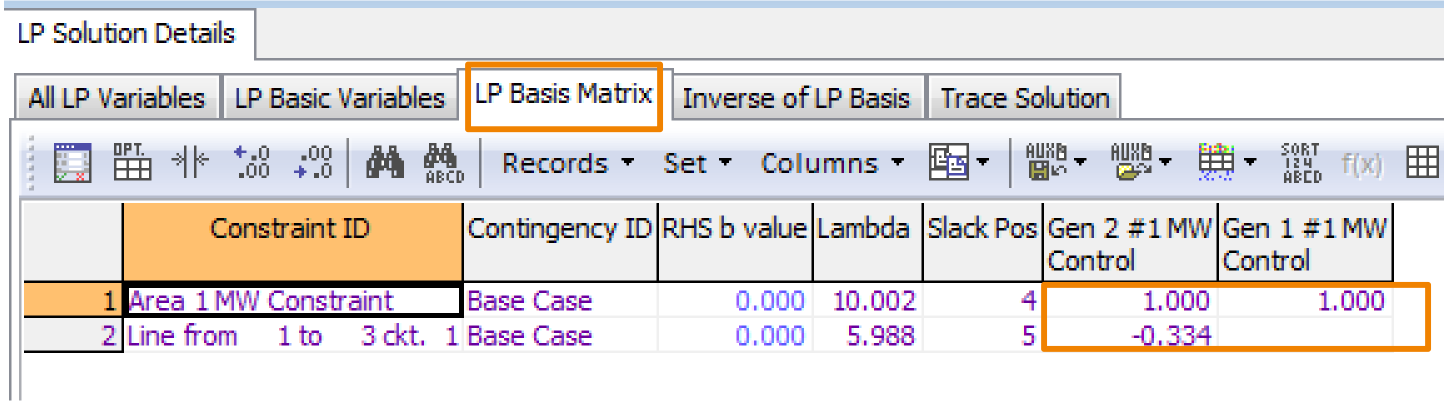
B3LP Revisited



B3LP Revisited

- Once the optimal dispatch has been determined, let us describe how the OPF calculates the LMPs.
- The first step is to calculate the marginal cost of the constraints.
- Go to the LP Solution Details -> LP Basis Matrix in the LP-OPF Dialog. The LP Basis Matrix contains the sensitivities of each constraint to each of the basic variables.

Can right click and choose “Show Full Tableau”



	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 2 #1 MW Control	Gen 1 #1 MW Control
1	Area 1 MW Constraint	Base Case	0.000	10.002	4	1.000	1.000
2	Line from 1 to 3 ckt. 1	Base Case	0.000	5.988	5	-0.334	

B3LP Revisited

- At the point of solution P_1 and P_2 are basic variables. P_3 is non-basic and hence zero.
- The values in the tableau correspond to the impact on the area total generation and to the PTDF sensitivities on line 1-3.
- Since Bus 1 is the slack bus in the system, the PTDF of line 1-3 with respect to a transfer from Bus 1 to the slack is zero.
- The PTDF of line 1-3 with respect to a transfer from Bus 2 to the slack (1) is –0.334.

B3LP Revisited

- The marginal cost of the area constraint is calculated by assuming that the area interchange increases in 1MW without changing the transmission line flow:

$$\mathbf{A}_B \rightarrow \begin{bmatrix}1 & 1\\ 0 & -0.334\end{bmatrix}\begin{bmatrix}\Delta P_1\\ \Delta P_2\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}\qquad \begin{array}{l}\text{1 Additional MW}\\ \text{No Additional Flow}\end{array}$$

$$\Rightarrow \begin{bmatrix}\Delta P_1\\ \Delta P_2\end{bmatrix}=\begin{bmatrix}1 & 1\\ 0 & -0.334\end{bmatrix}^{-1}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$$

- Increasing the Area Load in 1MW requires increasing Gen1 output in 1MW. Hence the Marginal Cost of the Area Constraint (Energy) is:

$$MC_E = \Delta P_1 \times IC_1 = 1 \times 10\ \$/\text{MWh} = 10\ \$/\text{MWh}$$

B3LP Revisited

- The marginal cost of the transmission line constraint is calculated by assuming that the total generation remains constant and the flow in the transmission line increases by 1MW.

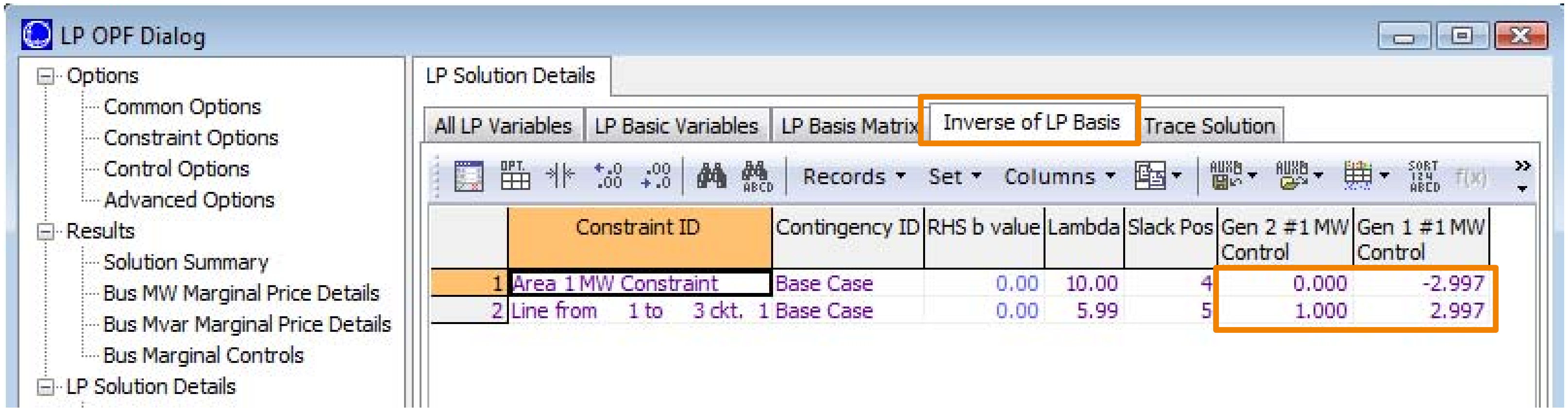
$$\begin{bmatrix} 1 & 1 \\ 0 & -0.334 \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -0.334 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

- Thus 1 additional MW of capacity allows increasing Gen1 output in 3MW and decreasing Gen2 output in 3MW. Hence the Marginal Cost of the Transmission Line Constraint (congestion) is:

$$MC_C = \Delta P_1 \times IC_1 + \Delta P_2 \times IC_2 = 3 \times 10 - 3 \times 12 = -6 \text{ \$/MWh}$$

B3LP Revisited

- Note that determining the marginal cost of a constraint involves obtaining the inverse of the basis matrix and obtaining the solution for a vector with a 1 in the row corresponding to the constraint.
- The solution is thus given by each column of the inverse of the basis matrix.



	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 2 #1 MW Control	Gen 1 #1 MW Control
1	Area 1 MW Constraint	Base Case	0.00	10.00	4	0.000	-2.997
2	Line from 1 to 3 ckt. 1	Base Case	0.00	5.99	5	1.000	2.997

B3LP Revisited

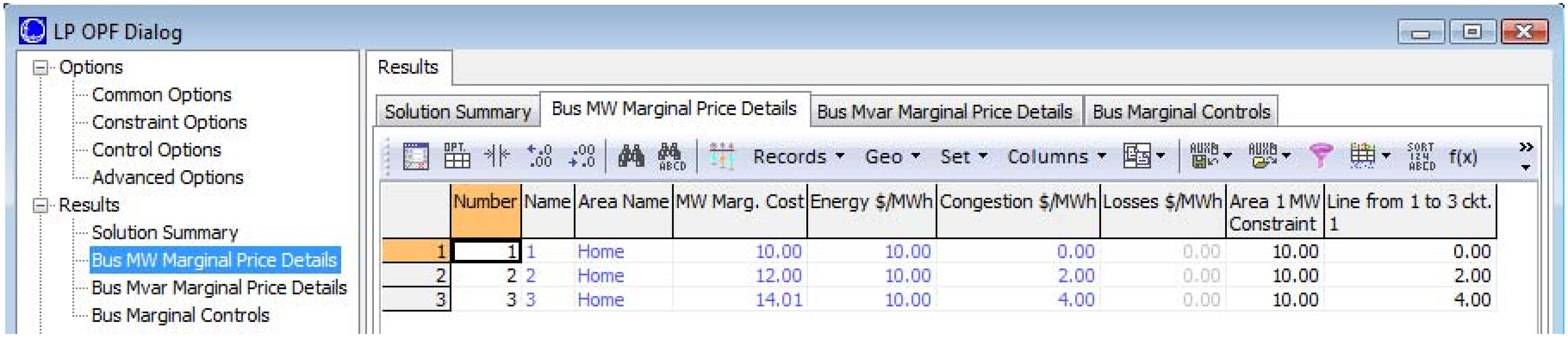
- Once the marginal costs of the constraints are determined, we can calculate the LMPs.
- For each bus, the LMP is equal to the sum of the sensitivities of the constraints with respect to transfers from each bus to the slack, times the marginal cost of the constraints.

$$LMP_i = \sum_{\substack{\text{Binding} \\ \text{Constraints } c}}^{C} Sens_{c,i} \times MC_c$$

B3LP Revisited

- This can be written using the transpose of the LP-Tableau

$$\begin{bmatrix} LMP_1 \\ LMP_2 \\ LMP_3 \end{bmatrix} = \mathbf{A}^T \begin{bmatrix} MC_E \\ MC_C \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -0.334 \\ 1 & -0.666 \end{bmatrix} \begin{bmatrix} 10 \\ -6 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 14 \end{bmatrix}$$



	Number	Name	Area Name	MW Marg. Cost	Energy $/MWh	Congestion $/MWh	Losses $/MWh	Area 1 MW Constraint	Line from 1 to 3 ckt. 1
1	1	1	Home	10.00	10.00	0.00	0.00	10.00	0.00
2	2	2	Home	12.00	10.00	2.00	0.00	10.00	2.00
3	3	3	Home	14.01	10.00	4.00	0.00	10.00	4.00