# Coordinated Initialization of the Load Distribution Equivalent, Load Characteristic, and Load Distributed Generation Models

NERC Load Modeling Task Force
January 12, 2016

Jamie Weber, Ph.D.
Director of Software Development

PowerWorld Corporation

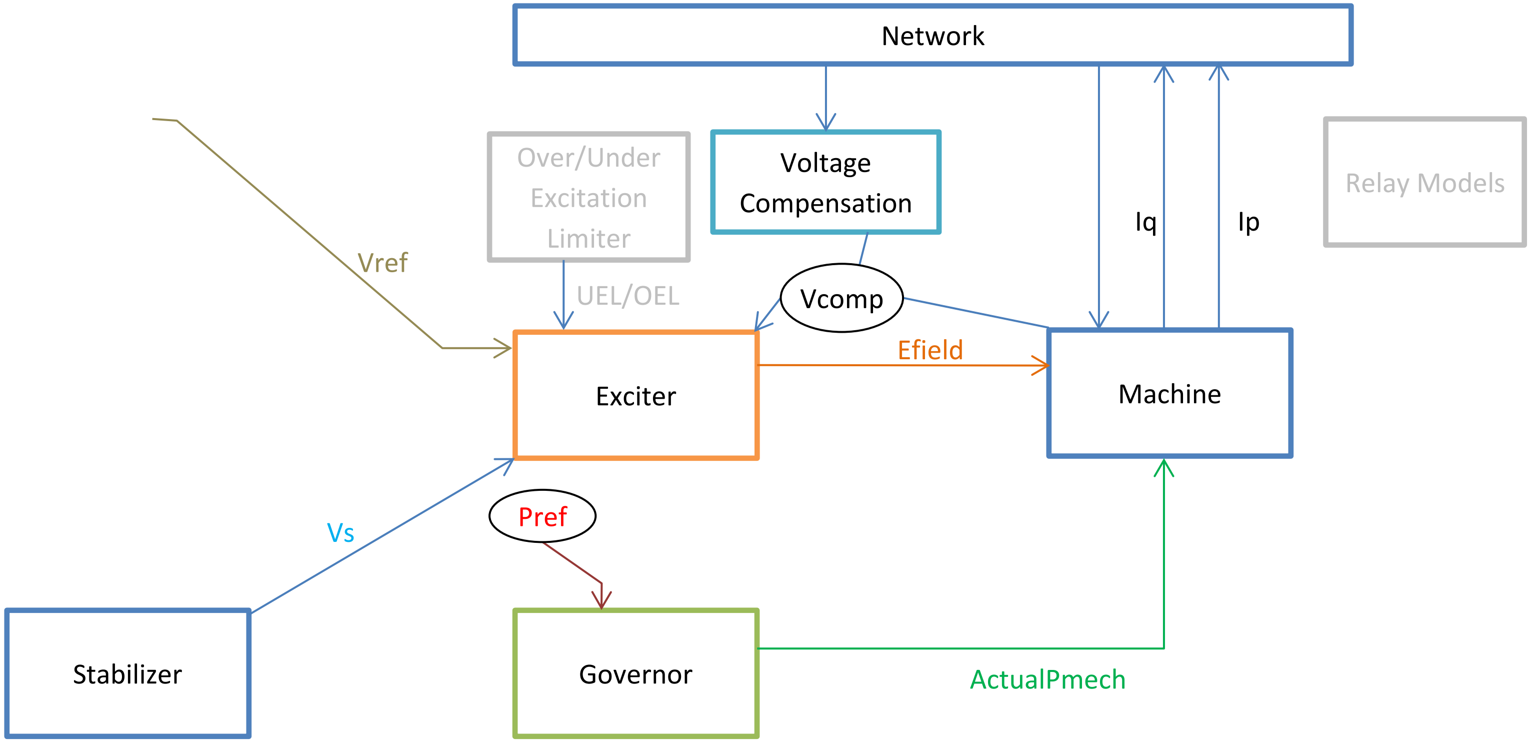
weber@powerworld.com
217 384 6330 ext 13

# Coordinated Initialization of Generator Models

- Generators have included a modular structure for several decades (1970s and onward)
- Synchronous generators needed up to 8 separate modules with coordinated initialization
  - Machine  (Generator/Converter Model)
  - Exciter (P and Q controller)
  - Governor (Drive Train)
  - Stabilizer (Pitch Control)
  - Under Excitation Limiter
  - Over Excitation Limiter
  - Compensator Model
  - Relay Model

# 8 Synchronous Generator Modules

- We have always done this with Generators

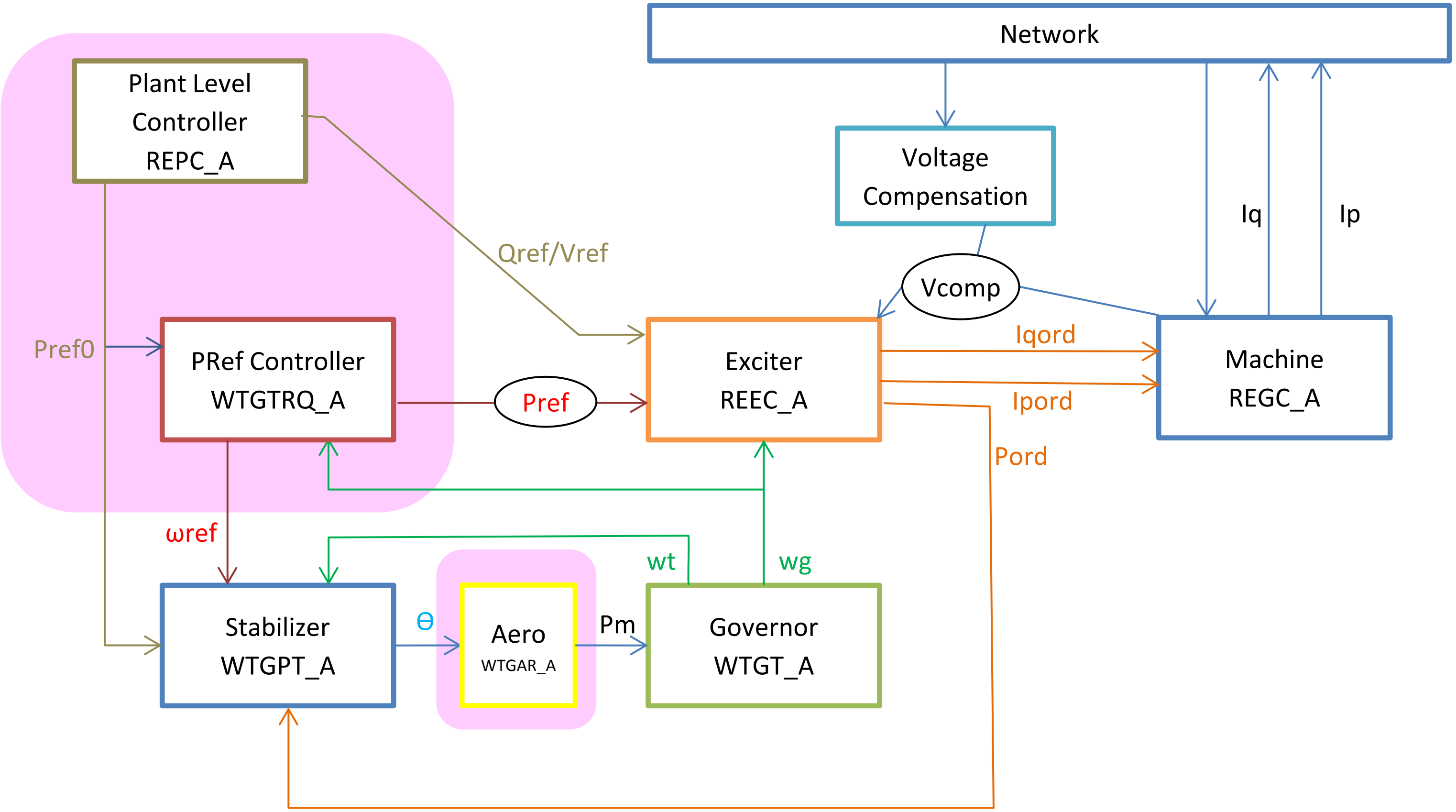


# Generator Modules Continue to Grow

- In past several years even more modules have been added
  - Aerodynamic Model (Type 3 Wind)
  - Pref Controller (Type 3 wind and LCFB1)
  - Plant Controller (renewable models)
  - AGC Controller (Implemented in Version 19 of PowerWorld Simulator)

# Type 3 Wind Turbine model added 3 new modules

- Pref Controller, Plant Controller, Aero



# Load Models have not kept up

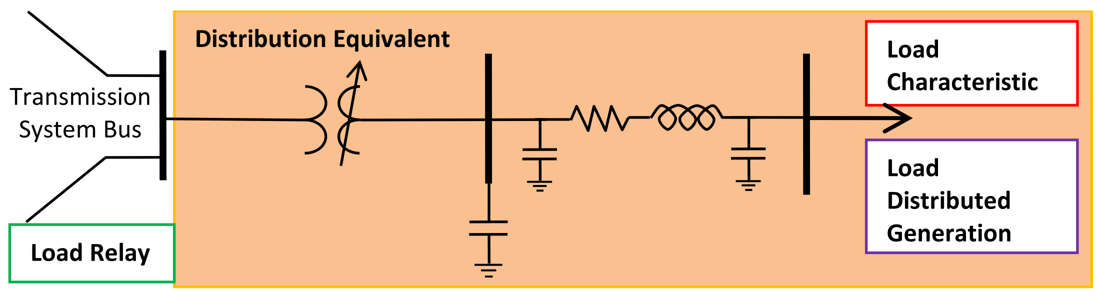
- Load Models have been stuck with only two modules
  - Load characteristic
  - Load relay
- The MOTORW model introduced in PSLF in the 1990s was a step in the right direction
  - MOTORW included a parameter indicating what percentage of the load was a motor
  - This meant we now had 3 modules
    - Dynamic Model
    - Algebraic Model
    - Relay Model
  - No longer required you to split the power flow load record to permit a load model split
- Relay model is always simple, but so is MOTORW
  - Does not require any coordination in the initialization of the models. Algebraic and Dynamic model just get split

# Initial Implementation of Distribution Equivalent

- Composite load model (CMPLDW) was designed within the WECC LMTF in the mid 2000s
  - The distribution equivalent is stuck inside the load characteristic
  - Has meant that new load models are gravitating toward being smashed into the CMPLDW framework
- Load Distributed Generation (Roof-top Solar for example)
  - WECC LMTF is now pushing us toward getting the Load Distributed Generation model out of the CMPLDW

# It is Time to Modularize: Just like Generators

- Natural for a load record to have 4 modules associated with transient stability
  - Load Characteristic (can also split algebraic/dynamic)
  - Relay Model
  - Distribution Equivalent
    - Added in PowerWorld Simulator Version 17 in January 2013
  - Load Distributed Generation Model
    - Added in PowerWorld Simulator Version 19 in November 2015




- Initialization of this model must be fully documented though

# Load Record: Distributed Generation

- Discussed in WECC LMTF for a few years and decision was made in 2014 to model with three new user input fields with each Load Record
  - **Dist MW Input**: the user entered MWs of distributed generation at the load
  - **Dist Mvar Input**: the user entered Mvars of distributed generation at the load
  - **Dist Status**: The status of the distributed generation (Open or Closed)
- Available in PowerWorld Simulator 19 now

| | Number of Bus | Name of Bus | Area Name of Load | Zone Name of Load | ID | Status | MW | Mvar | MVA | S MW | S Mvar | Dist Status | Dist MW Input | Dist Mvar Input | Dist MW | Dist Mvar | Net Mvar | Net MW |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | Two | Top | 1 | 1 | Closed | 80.00 | 20.00 | 82.46 | 80.00 | 20.00 | Closed | 40.00 | 0.00 | 40.000 | 0.000 | 20.000 | 40.000 |
| 2 | 3 | Three | Top | 1 | 1 | Closed | 220.00 | 40.00 | 223.61 | 220.00 | 40.00 | Open | 110.00 | 0.00 | 0.000 | 0.000 | 40.000 | 220.000 |
| 3 | 4 | Four | Top | 1 | 1 | Closed | 160.00 | 30.00 | 162.79 | 160.00 | 30.00 | Closed | 80.00 | 0.00 | 80.000 | 0.000 | 30.000 | 80.000 |
| 4 | 5 | Five | Top | 1 | 1 | Closed | 260.00 | 40.00 | 263.06 | 260.00 | 40.00 | Open | 130.00 | 0.00 | 0.000 | 0.000 | 40.000 | 260.000 |
| 5 | 6 | Six | Left | 1 | 1 | Closed | 400.00 | 0.00 | 400.00 | 400.00 | 0.00 | Closed | 200.00 | 0.00 | 200.000 | 0.000 | 0.000 | 200.000 |
| 6 | 7 | Seven | Right | 1 | 1 | Closed | 400.00 | 0.00 | 400.00 | 400.00 | 0.00 | Closed | 200.00 | 0.00 | 200.000 | 0.000 | 0.000 | 200.000 |

# Other Load Record Fields

- **Dist MW**, **Dist Mvar**: this is the actual MWs being seen by the power flow solution
  - This will be 0.0 if **DistStatus** = Open
  - This will be reduced if the voltage falls below the minimum voltage for constant power load
- **Net MW**: this is equal to the subtraction of the fields **MW – Dist MW**

| | Number of Bus | Name of Bus | Area Name of Load | Zone Name of Load | ID | Status | MW | Mvar | MVA | S MW | S Mvar | Dist Status | Dist MW Input | Dist Mvar Input | Dist MW | Dist Mvar | Net Mvar | Net MW |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | Two | Top | 1 | 1 | Closed | 80.00 | 20.00 | 82.46 | 80.00 | 20.00 | Closed | 40.00 | 0.00 | 40.000 | 0.000 | 20.000 | 40.000 |
| 2 | 3 | Three | Top | 1 | 1 | Closed | 220.00 | 40.00 | 223.61 | 220.00 | 40.00 | Open | 110.00 | 0.00 | 0.000 | 0.000 | 40.000 | 220.000 |
| 3 | 4 | Four | Top | 1 | 1 | Closed | 160.00 | 30.00 | 162.79 | 160.00 | 30.00 | Closed | 80.00 | 0.00 | 80.000 | 0.000 | 30.000 | 80.000 |
| 4 | 5 | Five | Top | 1 | 1 | Closed | 260.00 | 40.00 | 263.06 | 260.00 | 40.00 | Open | 130.00 | 0.00 | 0.000 | 0.000 | 40.000 | 260.000 |
| 5 | 6 | Six | Left | 1 | 1 | Closed | 400.00 | 0.00 | 400.00 | 400.00 | 0.00 | Closed | 200.00 | 0.00 | 200.000 | 0.000 | 0.000 | 200.000 |
| 6 | 7 | Seven | Right | 1 | 1 | Closed | 400.00 | 0.00 | 400.00 | 400.00 | 0.00 | Closed | 200.00 | 0.00 | 200.000 | 0.000 | 0.000 | 200.000 |

# Treatment of Distributed Generation in Power Flow

- Summary Information with Areas, Zones, Substations, etc…
  - Dist MW is separate summation from Load MW

| | Area Num | Area Name | AGC Status | Gen MW | Load MW | Dist MW | Shunt MW | Tot Sched MW | Int MW | ACE MW | Lambda | Loss MW |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | Top | ED | 367.42 | 720.00 | 120.000 | | 0.00 | -359.54 | -359.54 | 0.00 | 6.96 |
| 2 | 2 | Left | ED | 199.52 | 400.00 | 200.000 | | 0.00 | -200.82 | -200.82 | 0.00 | 0.34 |
| 3 | 3 | Right | ED | 400.99 | 400.00 | 200.000 | | 0.00 | 0.36 | 0.36 | 0.00 | 0.64 |

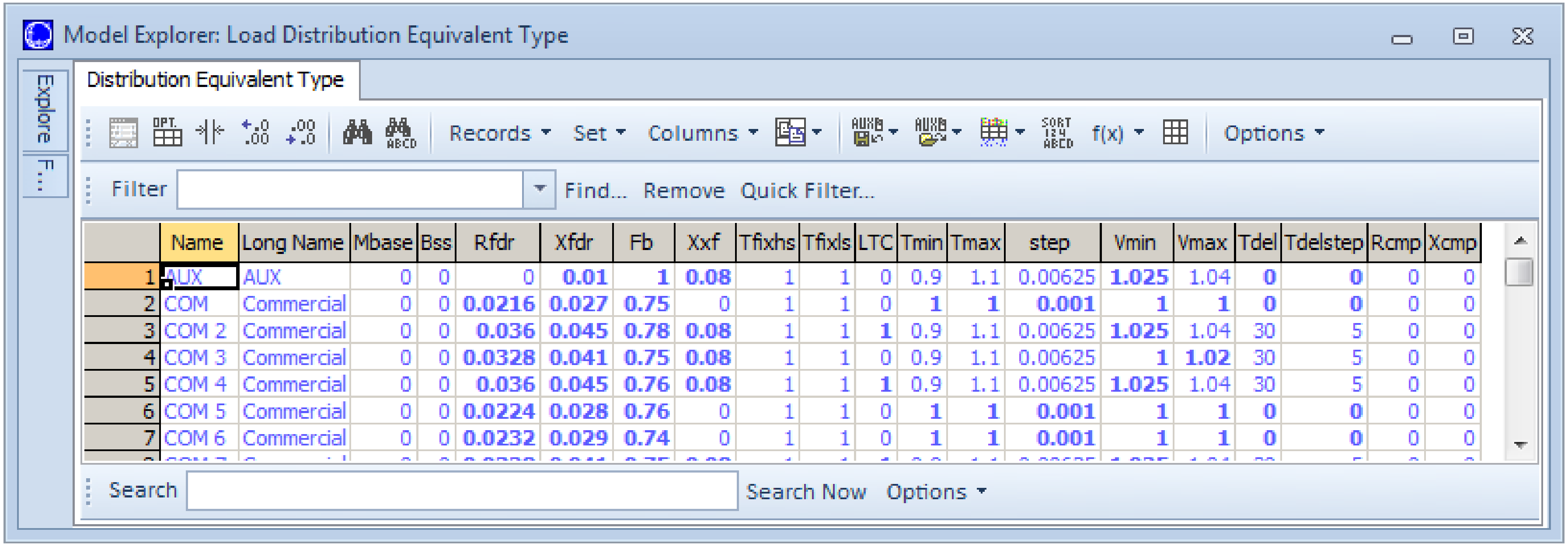
- Injection Group Treatment
  - Injected MW = Gen MW – Load MW – Dist Gen MW
- Contingency Actions
  - "Set, Change, Move" actions only act on Load portion
  - Open and Close actions also open the distributed gen

# Load Distribution Equivalent

- Supplementary model that defines an equivalent of the distribution system's transformer, capacitors, and feeder
- Created independently of the load characteristic models
- Can be used with any load characteristic model
- Design assumes small number of Load Distribution Equivalent Types with many different loads assigned to each

# Load Distribution Equivalent

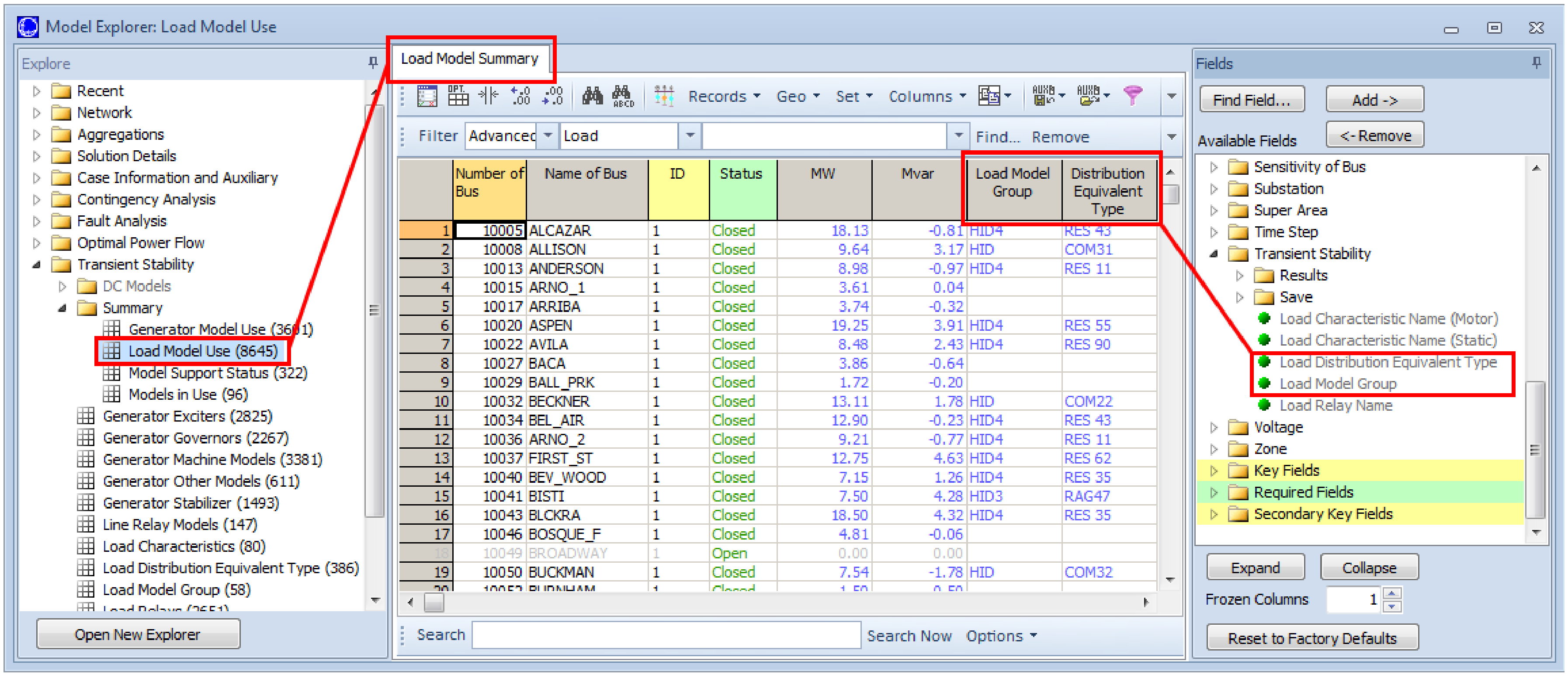
- First 17 parameters of the CMPLDW load characteristic model along with MVA base



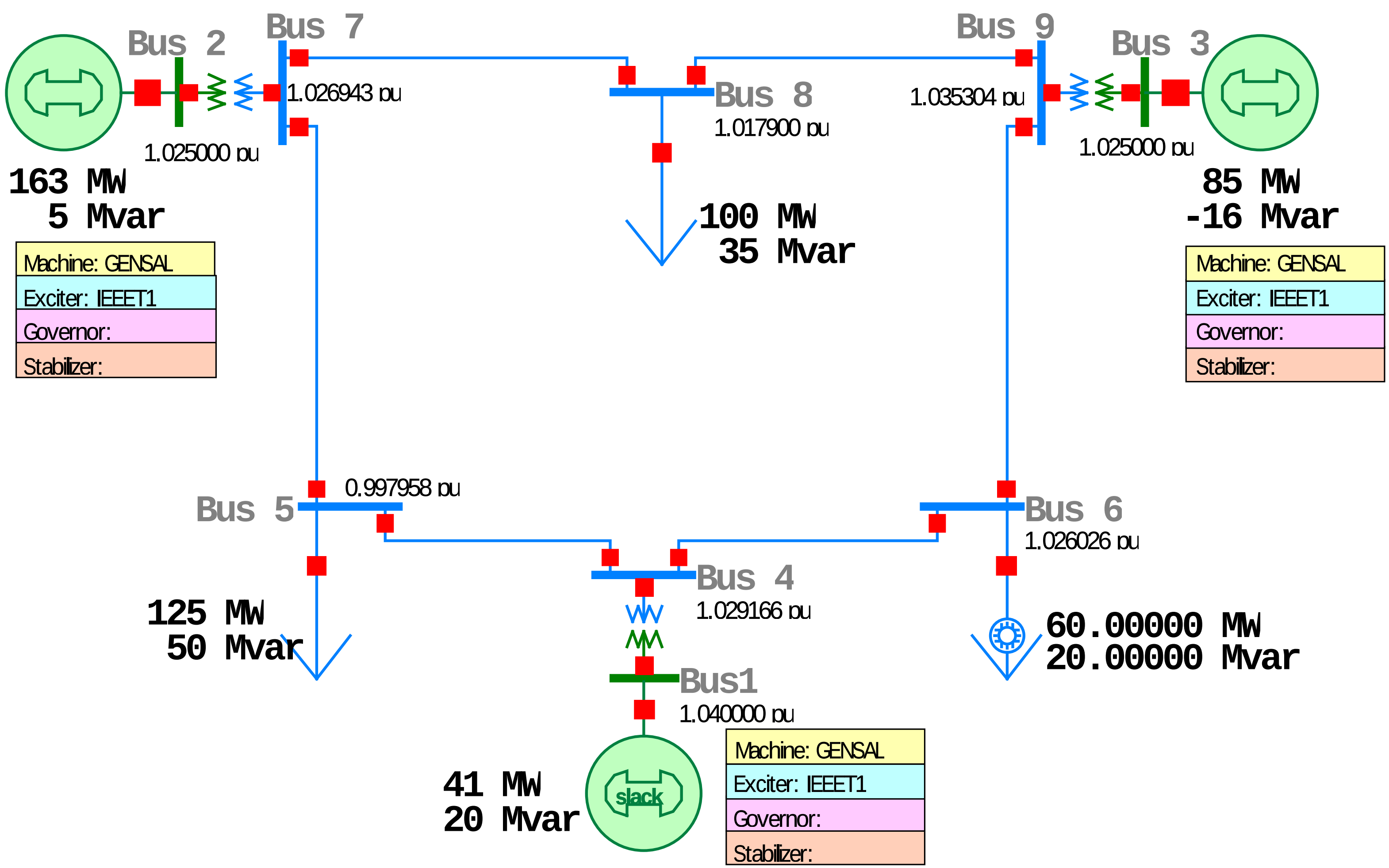

| | Name | Long Name | Mbase | Bss | Rfdr | Xfdr | Fb | Xxf | Tfixhs | Tfixls | LTC | Tmin | Tmax | step | Vmin | Vmax | Tdel | Tdelstep | Rcmp | Xcmp |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | AUX | AUX | 0 | 0 | 0 | 0.01 | 1 | 0.08 | 1 | 1 | 0 | 0.9 | 1.1 | 0.00625 | 1.025 | 1.04 | 0 | 0 | 0 | 0 |
| 2 | COM | Commercial | 0 | 0 | 0.0216 | 0.027 | 0.75 | 0 | 1 | 1 | 0 | 1 | 1 | 0.001 | 1 | 1 | 0 | 0 | 0 | 0 |
| 3 | COM 2 | Commercial | 0 | 0 | 0.036 | 0.045 | 0.78 | 0.08 | 1 | 1 | 1 | 0.9 | 1.1 | 0.00625 | 1.025 | 1.04 | 30 | 5 | 0 | 0 |
| 4 | COM 3 | Commercial | 0 | 0 | 0.0328 | 0.041 | 0.75 | 0.08 | 1 | 1 | 0 | 0.9 | 1.1 | 0.00625 | 1 | 1.02 | 30 | 5 | 0 | 0 |
| 5 | COM 4 | Commercial | 0 | 0 | 0.036 | 0.045 | 0.76 | 0.08 | 1 | 1 | 1 | 0.9 | 1.1 | 0.00625 | 1.025 | 1.04 | 30 | 5 | 0 | 0 |
| 6 | COM 5 | Commercial | 0 | 0 | 0.0224 | 0.028 | 0.76 | 0 | 1 | 1 | 0 | 1 | 1 | 0.001 | 1 | 1 | 0 | 0 | 0 | 0 |
| 7 | COM 6 | Commercial | 0 | 0 | 0.0232 | 0.029 | 0.74 | 0 | 1 | 1 | 0 | 1 | 1 | 0.001 | 1 | 1 | 0 | 0 | 0 | 0 |

# Assigning Load Distribution Equivalent Models

- Each Load record is assigned to a Distribution Equivalent



# What does this look like?

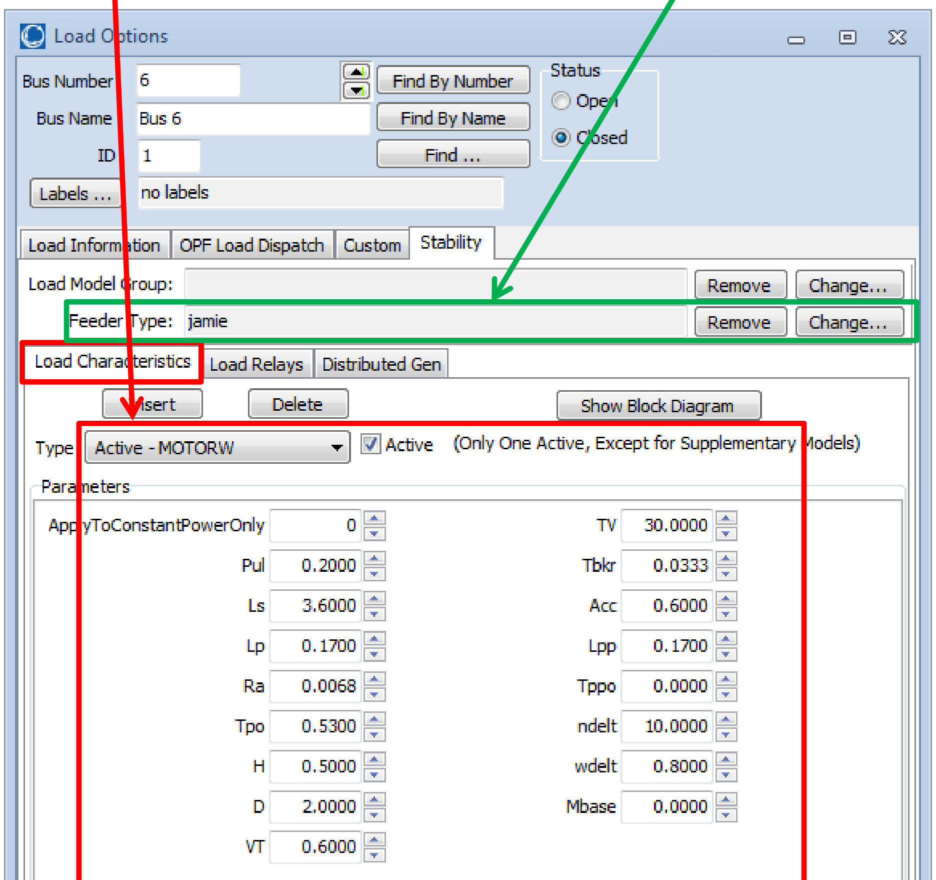


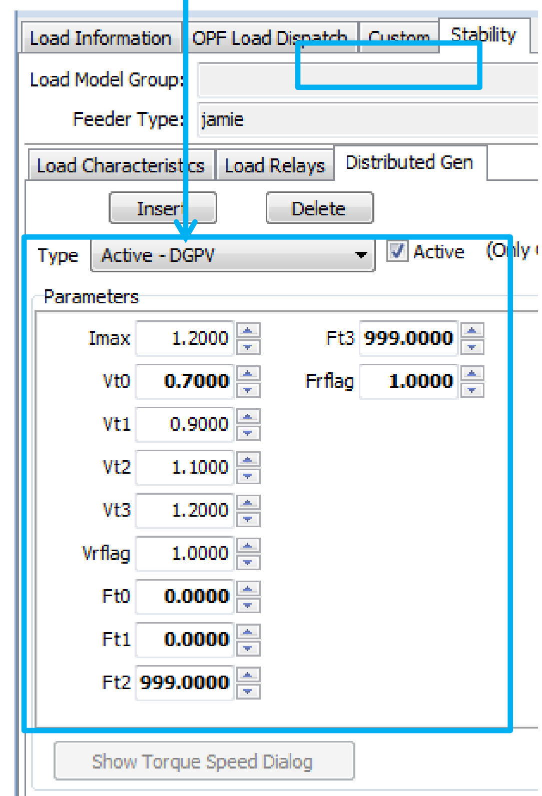
# Load Record has Modules

Load Characteristic

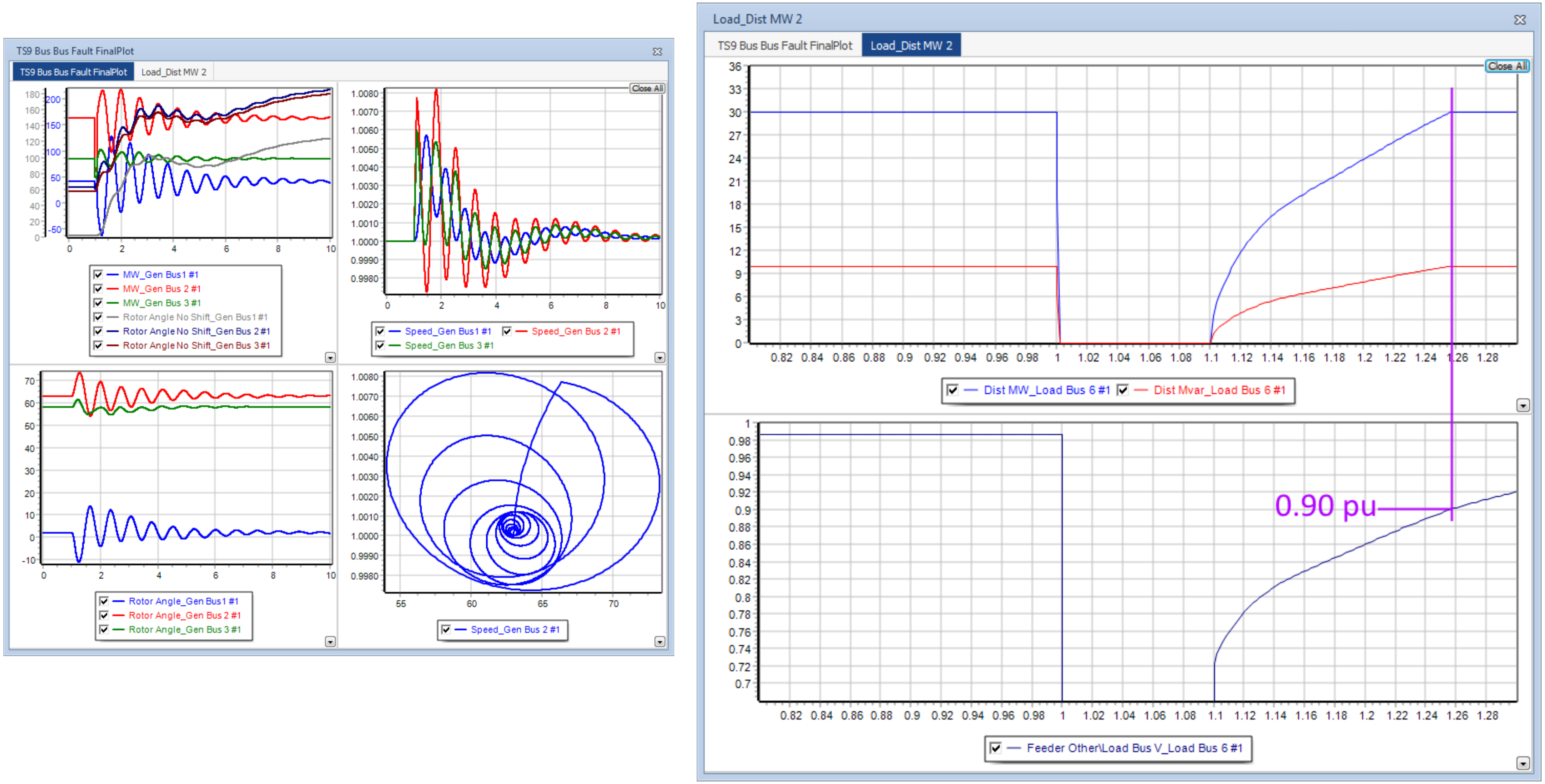
Distribution Equivalent

Distributed Generation



# Plot of Results: Right plot shows Distributed Gen



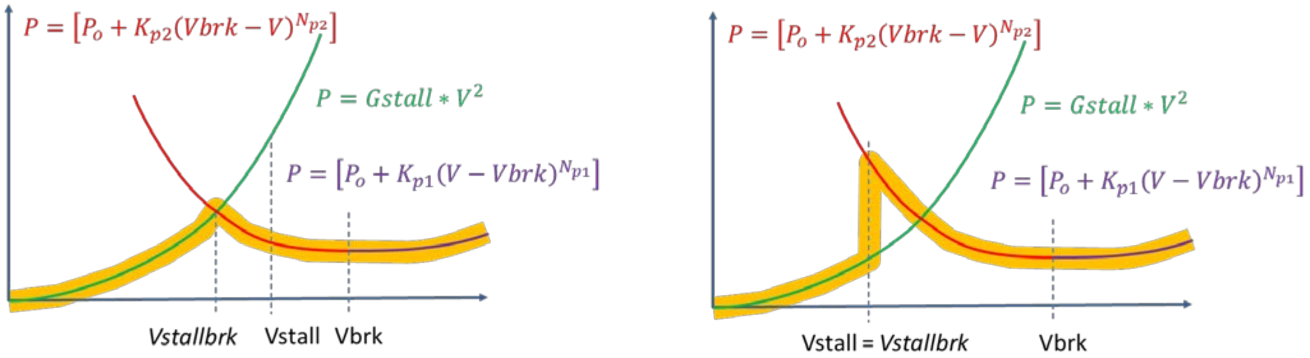
# Initialization

- When using a software model the fundamental details of the model should be known to the user
  - Equipment affects everyone as it is connected to a grid
  - Others need to model your equipment
  - This often means pseudo-code is needed
- Describing how a model is initialized is part of this
  - *Software vendor secret*: the hardest thing to do with transient stability models is to initialize them
  - Also some arbitrary decisions are sometimes made about a model during initialization → need pseudo-code
- Before we discuss coordinated initialization of these models → consider initialization of
  - Single Phase Air Conditioner → LD1PAC
  - Induction Motors → MOTORW, CIM5, MOTOR1, etc..

# LD1PAC Model: Algebraic Performance Model

- LD1PAC follows algebraic P/Q Curves
  - If *V* > *Vbrk* then
    - $P = \left[P_o + K_{p1}(V - Vbrk)^{N_{p1}}\right][1 + CmpKpf * \Delta f]$
    - $Q = \left[Q_o + K_{q1}(V - Vbrk)^{N_{q1}}\right][1 + CmpKqf * \Delta f]$
  - If *V* < *Vbrk* and *V* > *Vstallbrk* then
    - $P = \left[P_o + K_{p2}(Vbrk - V)^{N_{p2}}\right][1 + CmpKpf * \Delta f]$
    - $Q = \left[Q_o + K_{q2}(Vbrk - V)^{N_{q2}}\right][1 + CmpKqf * \Delta f]$
  - If *V* < *Vstallbrk* then
    - $P = Gstall * V^2$
    - $Q = Bstall * V^2$
- Important somewhat arbitrary decision
  - What is *Vstallbrk* ? Must fully document this stuff!

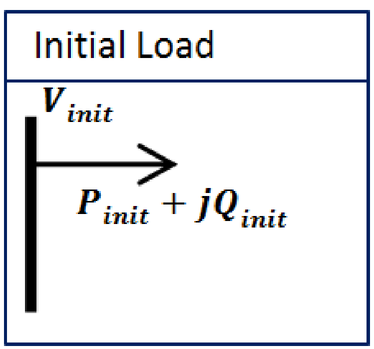
# $P_o$, $Q_o$, and Vstallbrk

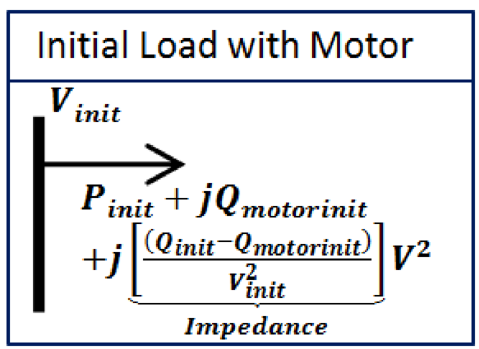


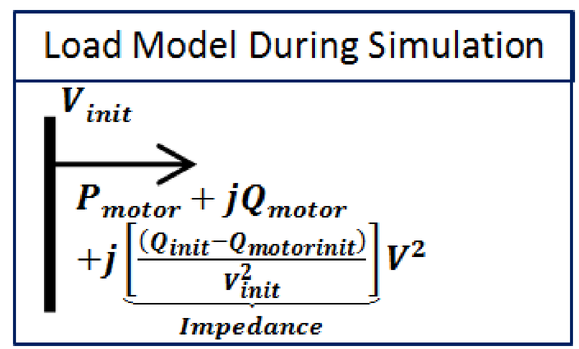

- $P_o = P_{init} - K_{p1}(V_{init} - Vbrk)^{N_{p1}}$
- *Vstallbrk* = intersection of the power stall curve and the power curve defined by Kp2 and Np2.
  - PowerWorld determines this to a tolerance of 0.0001 per unit voltage.
  - If this intersection is calculated as higher than *Vstall*, then instead set *Vstallbrk* = *Vstall* (this should be a very rare occurrence as it can result in strange results)
- $Q_o = P_{init}\left(\frac{\sqrt{1-CompPF^2}}{CompPF}\right) - K_{p1}(1.0 - Vbrk)^{N_{p1}}$
- As long as the motor is not stalled (Below *Vstall* for more than *Tstall* seconds), then the algebraic P and Q values following this yellow highlighted curves

# Induction Motor Initialization

- Mvar of an induction motor at initialization (steady state) is *<u>dependent</u>* on the terminal voltage and MW of the motor.
- There will be a mismatch between
  - Motor Mvar
  - Load Flow Record Initial Mvar
- Handled by including a shunt admittance as part of the load model to match the initial condition
  - These will be called ExtraMvars in this document



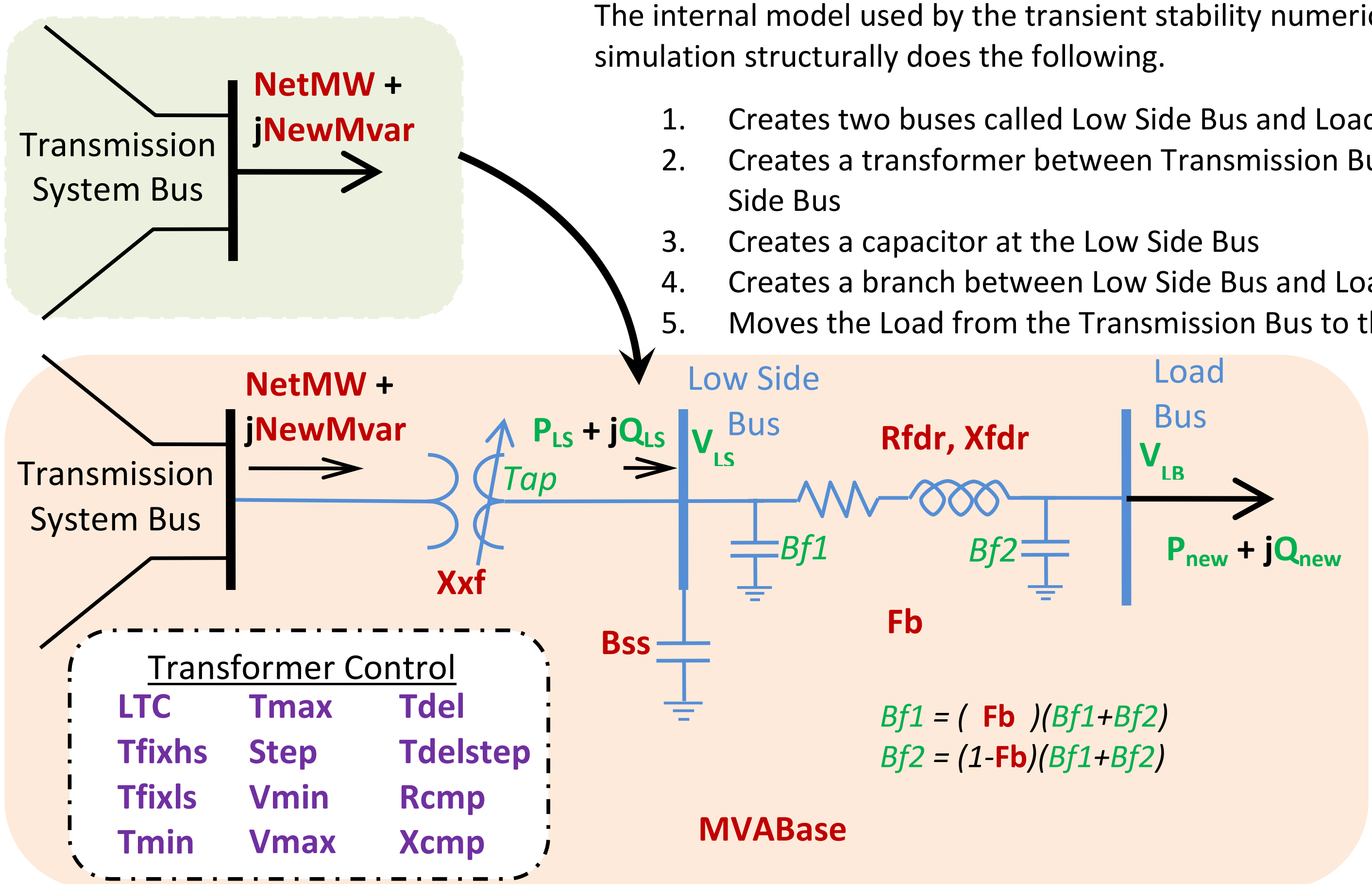



# Coordinated Initialization with Distribution Equivalent

The internal model used by the transient stability numerical simulation structurally does the following.

1. Creates two buses called Low Side Bus and Load Bus
2. Creates a transformer between Transmission Bus and Low Side Bus
3. Creates a capacitor at the Low Side Bus
4. Creates a branch between Low Side Bus and Load Bus
5. Moves the Load from the Transmission Bus to the Load Bus



# Important:
# Input Data Validation Check

- The **NetMW** and **NetMvar** of the load can potentially exceed the maximum power transfer of the Distribution Equivalent
  - Calculation of DistEquivMVABase
    - MVABase > 0 means DistEquivMVABase = MVABase
    - MVABase < 0 means DistEquivMVABase = Abs(**NetMW**/MVABase)
    - MVABase = 0 means DistEquivMVABase = **NetMW**/0.8
  - Notice that MVABase is a function of **NetMW** for MVABase <=0
- Problem for loads with extremely poor power factor
  - What if load is 1.2 MW and 30 Mvar? Impedances are based on base proportional to 1.2, but 30 Mvars across this may be too much.
- Software Solution: Add a validation error
  - Check if Estimate of Real Power $I^2R$ losses exceeds the **NetMW**
  - Clearly bad input data if

$$\underbrace{\left[\frac{NewMW^2 + NetMvar^2}{Vpu^2}\right]}_{Current\ Squared} * \underbrace{\left[Rfdr\frac{SystemMVABase}{DistEquivMVABase}\right]}_{R\ on\ system\ MVABase} > NetMW$$

# Important: Related Validation Check

- A lot of these weird validation errors actually occur with extremely small loads
  - How about MW = 0.001 and Mvar = 0.020
  - This kind of thing happens a lot when you get a real-time state estimation case → just noise from measurements and the state estimator solution
- PowerWorld Simulator has a hard-coded threshold
  - Any MW Load < 0.001 per unit (0.1 MW for 100 MVABase system) is never modeled with anything but an algebraic load model
  - Silly to model motors this small anyway
  - You will see warning messages inside Simulator indicating this is occurring

# Initialization Process

- Goal of Initialization is to calculate Tap, Bf1, Bf2, VLS, PLS, QLS, VLB, Pnew, and Qnew.
(Also might change **Bss**, **Rfdr**, and **Xfdr**)
- If you want the same results you need to define the rules precisely
  - It is quite likely that multiple values of Tap will get you inside the **Vmin** and **Vmax** range specified in the distribution equivalent
  - How you split Bf1, Bf2, and **Bss** might change results
  - How to reduce the **Rfdr** and **Xfdr** when the load bus voltage falls below 0.95 matters
- Define “precisely” → means psuedo-code

# Initialization Steps 1 – 2: Impedance Base Conversion

- Step 1: Calculation of DistEquivMVABase
  - MVABase > 0 means DistEquivMVABase = MVABase
  - MVABase < 0 means DistEquivMVABase = Abs(**NetMW**/MVABase)
  - MVABase = 0 means DistEquivMVABase = Abs(**NetMW**/0.8)
  - Note: This is a function of NetMW, so that means MW – DistMW of the distributed generation
- Step 2: Impedance parameters are given on this DistEquivMVABase base, so convert them to the SystemMVABase
  - **Xxf** = **Xxf** * *SystemMVABase*/ *DistEquivMVABase*.
    **Rfdr** = **Rfdr** * *SystemMVABase*/ *DistEquivMVABase*.
    **Xfdr** = **Xfdr** * *SystemMVABase*/ *DistEquivMVABase*.
    **Rcmp** = **Rcmp** * *SystemMVABase*/ *DistEquivMVABase*.
    **Xcmp** = **Xcmp** * *SystemMVABase*/ *DistEquivMVABase*.
    **Bss** = **Bss** / *SystemMVABase** *DistEquivMVABase*.

# Initialization Steps 3 – 4: Transformer Setup

- Step 3: Convert Transformer Tap values and impedances to the SystemMVABase
  - Variable tab is on the low side bus
  - **Xxf** = **Xxf** * (**Tfixhs**)$^2$
  - **Step** = **Step**/**Tfixhs**
  - **Tmin** = (**Tmin** + **Tfixls** - 1)/**Tfixhs**
  - **Tmax** = (**Tmax** + **Tfixls** - 1)/**Tfixhs**
- Step 4: Set tap ratio (Tap) needed.
  - Sending end flow is Net values so (Load – DistGen)
  - Calculate exact tap ratio needed to give Low Side Bus Voltage of (**Vmin** + **Vmax**)/2 (arbitrary decision)
    - See Section 3.2.1 of companion PDF document for exact equations
  - Round to nearest discrete step and enforce **Tmin** and **Tmax**

There are likely a few Tap values which get you inside Vmin and Vmax

# Initialization Steps 5 – 7:

- Step 5: Calculate the Low Side Bus Voltage (VLS) and the Low Side Bus P and Q flow *exactly* (PLS, QLS)
  - See Section 3.2.2 of companion PDF document for exact equations
- Step 6: Initialize Bf1 and Bf2 to zero
- Step 7: If VLS < 0.95 then automatically set **Rfdr** and **Xfdr** to minimum value
  - **Rfdr** = 0.0000001 per unit
  - **Xfdr** = 0.00001 per unit

- This is where things get complicated
  - Calculation Load Bus Voltage (VLB) depends on Bf1
  - As mentioned in induction motor initialization, there are ExtraVars that come from that initialization which depends on VLB
  - The distribution equivalent model specifies that these ExtraVars be split between the from and to end of the feeder according to **Fb** input option
    - *Bf1 = (* **Fb** *)(Bf1 + Bf2)*
    - *Bf2 = (1-***Fb***)(Bf1 + Bf2)*
  - But Bf1 is used to calculate VLB

If Fb = 0, things are a LOT easier!

# Initialization Steps 8 – 9

- Step 8: Using present values of Bf1 and Bf2, estimate both the Load Bus Voltage (VLB) and the flow reaching the Load Bus (Pnew, Qnew)
  - See Section 3.2.2 of companion PDF document for exact equations
- Step 9: If magnitude of VLB < 0.95 then the feeder impedances are reduced by a factor such that VLB = 0.95 (exactly) and update Pnew, Qnew
  - See Section 3.2.3 of companion PDF document for exact equations

# Initialization Steps 10 – 11

- Step 10: Using Values of VLB, Pnew, Qnew initialize the dynamic load characteristic models
  - If Distributed Generation Model is present, then the Load Characteristic Models will use (Pnew + DistMW) and (Qnew + DistMvar)
  - Part of Load Characteristic initialization will result in ExtraMvars
- Step 11: If we have reduced **Rfdr** and **Xfdr** to minimum value already, then Exit Initialization and leave ExtraMvars with Load Bus

# Initialization Steps 12
# ExtraMvars → Bf1, Bf2

- Step 12: Allocate ExtraMvars to Bf1 and Bf2
  - If (ExtraMvars < 1E-4 per unit) OR (**Fb** < 0.001), then stick them all at the Load Bus
    - *Bf2* = *Bf2* + ExtraMvars/($VLB^2$)
    - *ExitShortly* = True
  - Else
    - *Bf1* = *Bf1* + **Fb***ExtraMvars/($VLS^2$)
    - *Bf2* = *Bf2* + (1 – **Fb**)*ExtraMvars/($VLB^2$)
    - *ExitShortly* = False

# Initialization Steps 13: Coordinate Bf1, Bf2 with **Bss**

- Step 13: If *Bf1* and *Bf2* are negative and **Bss** > 0 then reduce **Bss** toward zero to cancel out *Bf1* and *Bf2*.
  - *ExitShortly* = False
  - Bf1var = *Bf1* * VLS $^2$
  - Bf2var = *Bf2* * VLB $^2$
  - Bssvar = **Bss** * VLS $^2$
  - If abs( Bf1var + Bf2Var ) > Bssvar then
    - tempVar = BssVar
    - **Bss** = 0

    **// Set Bss=0 and reduce magnitude of Bf1 and Bf2**
  - If **Fb** = 1 then
    - tempVar = abs(Bf1Var)
    - **Bss** = **Bss** – tempVar / VLS $^2$

    **// Reduce Bss by Bf1Var and reallocate Mvars to *Bf1* and *Bf2***
  - Else
    - tempVar = abs(Bf2var) / (1 – **Fb**)
    - **Bss** = **Bss** – tempVar / VLS $^2$

    **// attempt to push Bf2 toward zero**
  - *Bf1* = *Bf1* + **Fb** * tempVar / VLS $^2$
  - *Bf2* = *Bf2* + (1 – **Fb**) * tempVar / VLS $^2$

# Initialization Steps 14: Finish Off

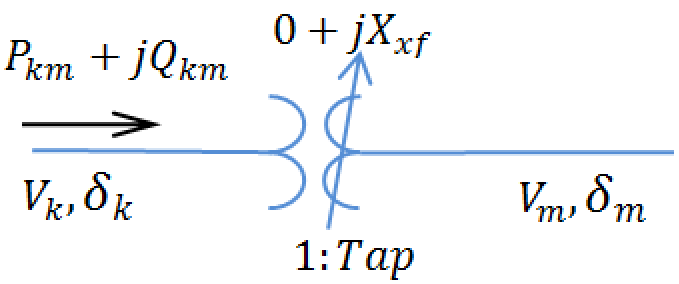
- Step 14:
  - If (ExitShortly) then Exit Initialization
    Else go back to Step 8 and repeat

# Summary

- Transparency of stability models is vital if you want to share models
  - Need good documentation
  - Often need some psuedo-code as block diagrams aren't enough
    - Various ways to implement non-windup PI limits
    - Algebraic models get weird (Bf1, Bf2, **Bss** coordination of distribution equivalent)
    - Calculation of *Vstallbrk* on LD1PAC
- Good examples of how to share model specifications
  - H6B governor model from John Undrill and implemented in PSLF https://www.wecc.biz/Reliability/H6b-Governor-Model-Specification.pdf
    - I implemented this in PowerWorld Simulator in February 2014 very quickly because documentation was excellent
    - Psuedo-code was important because of unique non-windup PI limit
  - Colstrip Acceleration Trend Relay from Jamie Weber implemented in PowerWorld Simulator http://www.powerworld.com/WebHelp/Content/TransientModels_PDF/Generator/Others/Relay%20Model%20ATRRELAY.pdf
  - Generic Wind and Solar Models
    - However, input units on which MVABase is still not clear on a few of these models…
    - Psuedo-code would have made that more clear

# Tap Calculation Equation (Section 3.2.1 of PDF)

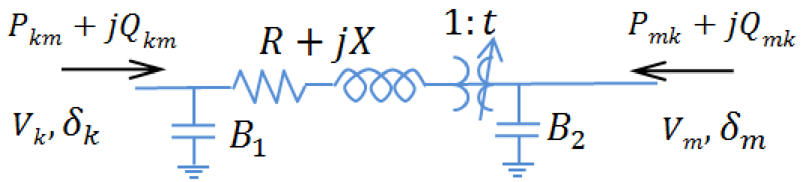
- Initial condition and the input parameters $P_{km}$, $Q_{km}$, $V_k$, and $X_{xf}$.




- We use $V_m = \frac{V_{min}+V_{max}}{2}$
- PDF document shows derivation of required $Tap$.
- $Tap = \sqrt{\frac{(V_k V_m)^2}{(Q_{km}X_{xf}-V_k^2)^2+(X_{xf}P_{km})^2}}$

# Calculation of Far Bus Complex Voltage and PQ Flows (Section 3.2.2)

- General Network as Follows




$$g_{kk} = \frac{R}{(R^2+X^2)} \qquad b_{kk} = \frac{-X}{(R^2+X^2)} + B_1 \qquad g_{km} = g_{mk} = \frac{-R}{(R^2+X^2)t}$$

$$g_{mm} = \frac{R}{(R^2+X^2)t^2} \qquad b_{mm} = \frac{-X}{(R^2+X^2)t^2} + B_2 \qquad b_{km} = b_{mk} = \frac{X}{(R^2+X^2)t}$$

- Calculate Complex Vm

$$\begin{bmatrix} e_k g_{km} + f_k b_{km} & | & -e_k b_{km} + f_k g_{km} \\ f_k g_{km} - e_k b_{km} & | & -f_k b_{km} - e_k g_{km} \end{bmatrix} \begin{bmatrix} e_m \\ f_m \end{bmatrix} = \begin{bmatrix} P_{km} - e_k^2 g_{kk} - f_k^2 g_{kk} \\ Q_{km} + f_k^2 b_{kk} + e_k^2 b_{kk} \end{bmatrix}$$

- Calculate far end flows

$$P_{mk} = +e_m^2 g_{mm} + e_m e_k g_{mk} - e_m f_k b_{mk} + f_m^2 g_{mm} + f_m f_k g_{mk} + f_m e_k b_{mk}$$
$$Q_{mk} = -f_{km}^2 b_{mm} + f_m e_k g_{mk} - f_m f_k b_{mk} - e_m^2 b_{mm} - e_m f_k g_{mk} - e_m e_k b_{mk}$$

# Section 3.2.2 Admittance Values Transformer and Feeder Branch

For our Transformer branch the admittance parameters are as follows.

$$g_{kk} = 0 \qquad b_{kk} = -\frac{1}{X_{xf}} \qquad g_{km} = g_{mk} = 0$$

$$g_{mm} = 0 \qquad b_{mm} = -\frac{1}{X_{xf}Tap^2} \qquad b_{km} = b_{mk} = \frac{1}{X_{xf}Tap}$$

For our Feeder branch the admittance parameters are as follows.

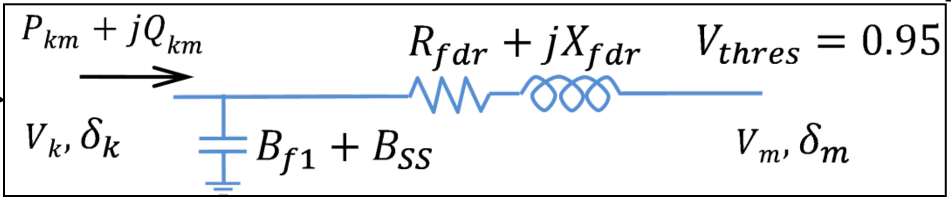
(Note: the capacitors are lumped into the feeder admittances for calculation purposes here.)

$$g_{kk} = \frac{R_{dfr}}{R_{fdr}^2 + X_{fdr}^2} \qquad b_{kk} = \frac{-X_{dfr}}{R_{fdr}^2 + X_{fdr}^2} + B_{f1} + B_{ss} \qquad g_{km} = g_{mk} = \frac{-R_{dfr}}{R_{fdr}^2 + X_{fdr}^2}$$

$$g_{mm} = \frac{R_{dfr}}{R_{fdr}^2 + X_{fdr}^2} \qquad b_{mm} = \frac{-X_{dfr}}{R_{fdr}^2 + X_{fdr}^2} + B_{f2} \qquad b_{km} = b_{mk} = \frac{X_{dfr}}{R_{fdr}^2 + X_{fdr}^2}$$

# Calculation of Far Bus Values with a Voltage Constraint

- Feeder Network at this point is




- Group the Bss and Bf1 terms in Q flow and define

$$g_{kk} = \frac{R_{dfr}}{R_{fdr}^2 + X_{fdr}^2} \qquad b_{kk} = \frac{-X_{dfr}}{R_{fdr}^2 + X_{fdr}^2}$$

$$g_{km} = \frac{-R_{dfr}}{R_{fdr}^2 + X_{fdr}^2} \qquad b_{km} = \frac{X_{dfr}}{R_{fdr}^2 + X_{fdr}^2}$$

- The solve following three nonlinear equations using Newton's method

$$P_{km} - MULT\begin{pmatrix} +e_k^2 g_{kk} + e_k e_m g_{km} - e_k f_m b_{km} \\ +f_k^2 g_{kk} + f_k f_m g_{km} + f_k e_m b_{km} \end{pmatrix} = 0$$

$$[Q_{km} + (B_{SS} + B_{f1})V_k^2] - MULT\begin{pmatrix} -f_k^2 b_{kk} + f_k e_m g_{km} - f_k f_m b_{km} \\ -e_k^2 b_{kk} - e_k f_m g_{km} - e_k e_m b_{km} \end{pmatrix} = 0$$

$$e_m^2 + f_m^2 - V_{thres}^2 = 0$$

- Convergence Tolerance of
  - P and Q equation 1E-5
  - Voltage equation 1E-8
- Jacobian Matrix

$$\begin{bmatrix} -MULT\begin{pmatrix} e_k g_{km} \\ +f_k b_{km} \end{pmatrix} & -MULT\begin{pmatrix} -e_k b_{km} \\ +f_k g_{km} \end{pmatrix} & -\begin{pmatrix} +e_k^2 g_{kk} + e_k e_m g_{km} - e_k f_m b_{km} \\ +f_k^2 g_{kk} + f_k f_m g_{km} + f_k e_m b_{km} \end{pmatrix} \\ -MULT\begin{pmatrix} f_k g_{km} \\ -e_k b_{km} \end{pmatrix} & -MULT\begin{pmatrix} -f_k b_{km} \\ -e g_{km} \end{pmatrix} & -\begin{pmatrix} -f_k^2 b_{kk} + f_k e_m g_{km} - f_k f_m b_{km} \\ -e_k^2 b_{kk} - e_k f_m g_{km} - e_k e_m b_{km} \end{pmatrix} \\ 2e_m & 2f_m & 0 \end{bmatrix}$$