Derivation of the GENTPF and GENTPJ models

WECC MVWG
November 18, 2015

Jamie Weber, Ph.D.
Director of Software Development
weber@powerworld.com
217 384 6330 ext 13

PowerWorld Corporation

2001 South First Street
Champaign, Illinois 61820
+1 (217) 384.6330

support@powerworld.com
http://www.powerworld.com

Outline

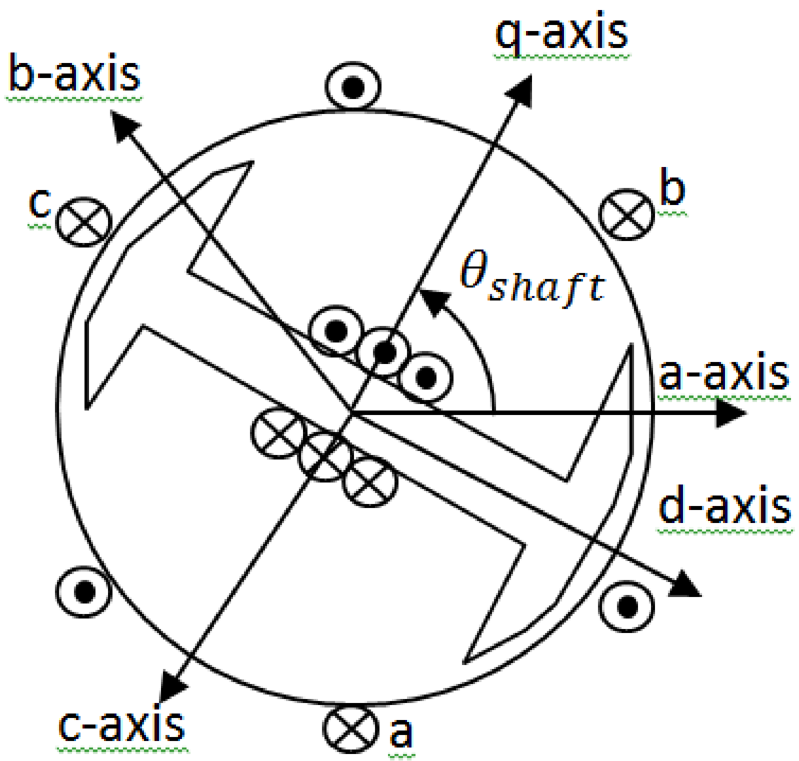
- GENROU/GENSAL models
  - References
  - Network Boundary Equation
  - Treatment of Saturation
- GENTPF/GENTPJ models
  - References
  - John Undrill equations from 2012 result in GENTPF/GENTPJ models
  - Derivation of GENTPF/GENTPJ starting from GENROU
- Implications of GENTPF/GENTPJ model
- More format write up with details is on PowerWorld's website at
  - http://www.powerworld.com/files/GENROU-GENSAL-GENTPF-GENTPJ.pdf

First Consider the GENROU/GENSAL Models

- GENROU model (and GENSAL) has been around since the infancy of transient stability analysis
- Classic book references I have found
  - Charles Concordia, *Synchronous Machine*, John Wiley & Sons, 1951
  - E. Kimbark, *Power System Stability: Synchronous Machines*, Dover Publications, Inc, 1956
  - William Lewis, *The Principles of Synchronous Machines*, 1959
  - P. Kundar, *Power System Stability and Control*, McGraw-Hill, 1994
  - P. Anderson, A. Fouad, *Power System Stability and Control*, IEEE Press, 1994
- My personal favorite reference
  - Provides a derivation from basic physics with all the assumptions that get to GENROU/GENSAL equations
  - P. Sauer, M.A. Pai, *Power System Dynamics and Stability*, Prentice Hall , 1998

Aside about d/q axis and rotor angle

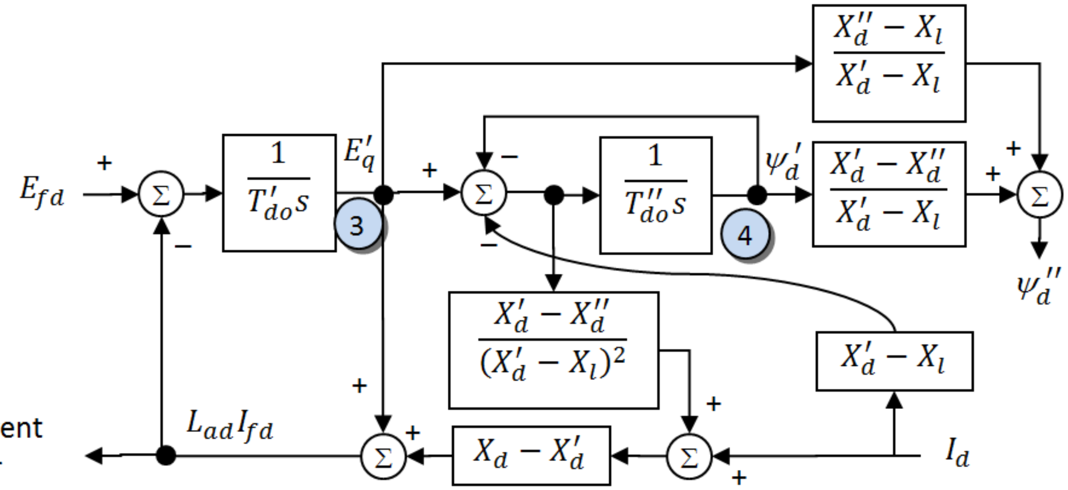
- Be careful when looking at all these references
  - Comparing equations between references is very hard → lots of sign differences
- d-axis is determined by right-hand rule on rotor
- Choice of q axis
  - 90 degrees *leading* d-axis
    - PowerWorld/PSLF/PSS/E choice
    - Sauer/Pai book page (page 25)
    - Kundar book (page 46)
  - 90 degrees *lagging* q-axis
    - Anderson/Fouad book (page 84)
- Choice of rotor angle
  - Angle behind the q-axis
    - PowerWorld/PSLF/PSS/E choice
    - Sauer/Pai book page (page 25)
  - Angle behind the d-axis
    - Anderson/Fouad book (page 84)
    - Kundar book (page 46)

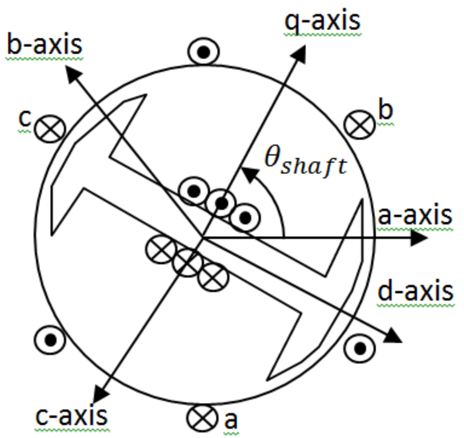


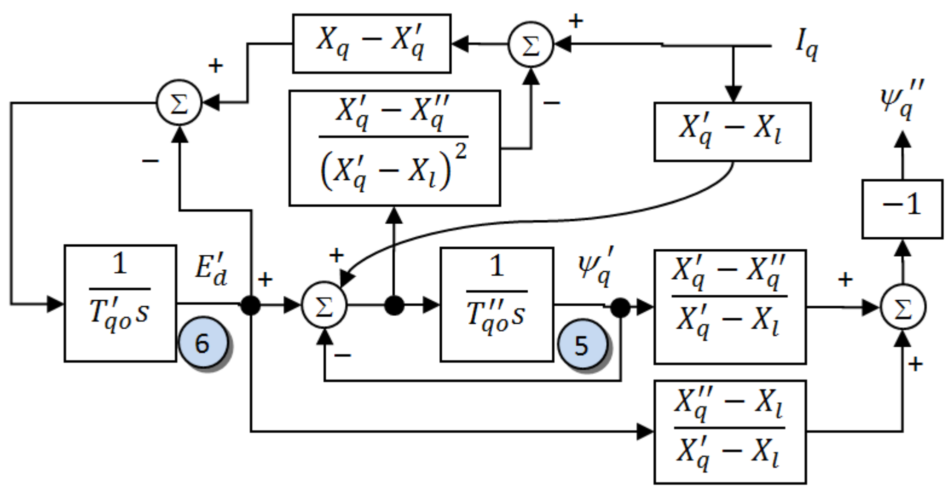
GENROU/GENSAL Derivation

- A fundamental derivation of a GENROU and GENSAL models can be found in Chapter 3 of the book *Power System Dynamics and Stability* by Peter Sauer and M.A. Pai from 1998.
  - Derivation starts from first principals represented by equations 3.1 – 3.9 on page 24 – 25
  - Culminates in Equations 3.148 – 3.159 on page 42.
  - Page 42 exactly represents GENROU and GENSAL *without saturation* (with a minor sign difference in one term)

GENROU without Saturation







Mechanical Differential Equations

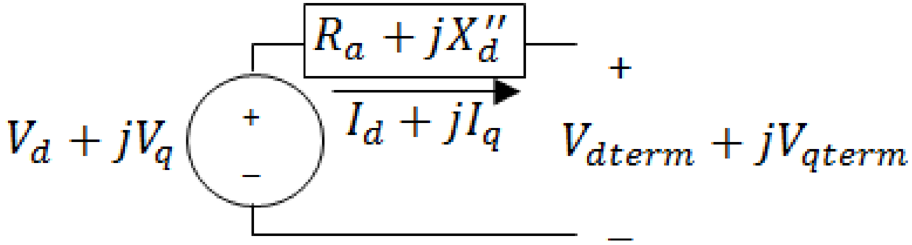
1. $$\dot{\delta} = \omega * \omega_0$$
2. $$\dot{\omega} = \frac{1}{2H}\left(\frac{P_{mech} - D\omega}{1+\omega} - T_{elec}\right)$$

- *P_{mech} = mechanical power which is an input from the governor model*
- *H and D are inputs to the model*
- $T_{elec} = \psi_d I_q - \psi_q I_d$
  - $\psi_q = \psi_q'' - I_q X_d''$
  - $\psi_d = \psi_d'' - I_d X_d''$
- ω = per unit speed deviation
  - ω=0 means we are at synchronous speed
  - ω=1 would mean it's spinning at double synchronous speed
- ω_0 = synchronous speed $2\pi f_0$
  - f_0 is the nominal system frequency in Hz

GENROU without Saturation Network Boundary Equation Interface

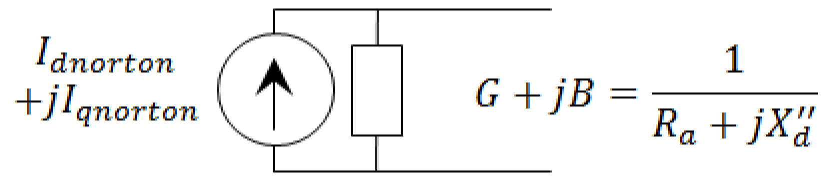
- Can be modeled as a circuit with a voltage source with Thevenin impedance
  - $\psi''(\boldsymbol{t}) = |\psi''|\boldsymbol{e}^{\boldsymbol{j}[(1+\omega)\boldsymbol{t}-\boldsymbol{\alpha}]} = \psi_d'' + j\psi_q''$ (sinusoid)
  - $\frac{\boldsymbol{d}\psi(\boldsymbol{t})}{\boldsymbol{dt}} = \boldsymbol{j}(1+\omega)|\psi''|\boldsymbol{e}^{\boldsymbol{j}[(1+\omega)\boldsymbol{t}-\boldsymbol{\alpha}]}$
  - $\mathbf{V} = \frac{\boldsymbol{d}\psi(\boldsymbol{t})}{\boldsymbol{dt}} = j(1+\omega)\left(\psi_d'' + j\psi_q''\right)$
  - $V_d + jV_q = \left(-\psi_q'' + j\psi_d''\right)(1+\omega)$
  - $Z_{source} = R_a + jX_d''$

Modeled as a standard circuit equation!

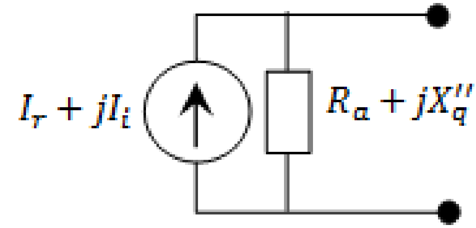


GENROU without Saturation Network Boundary Equation Interface

- Convert this to a Norton
  - $Y_{source} = \frac{1}{R_a + jX''_d} = G + jB$
  - $I_{dnorton} + jI_{qnorton} = (V_d + jV_q)(G + jB)$



- Convert current to network reference frame
  - $I_r + jI_i = (I_{dnorton} + jI_{qnorton})e^{j(\delta - \frac{\pi}{2})}$
  - Multiply by complex number using rotor angle (see writeup on website for details of network transformation)



Modeled as a standard circuit equation!

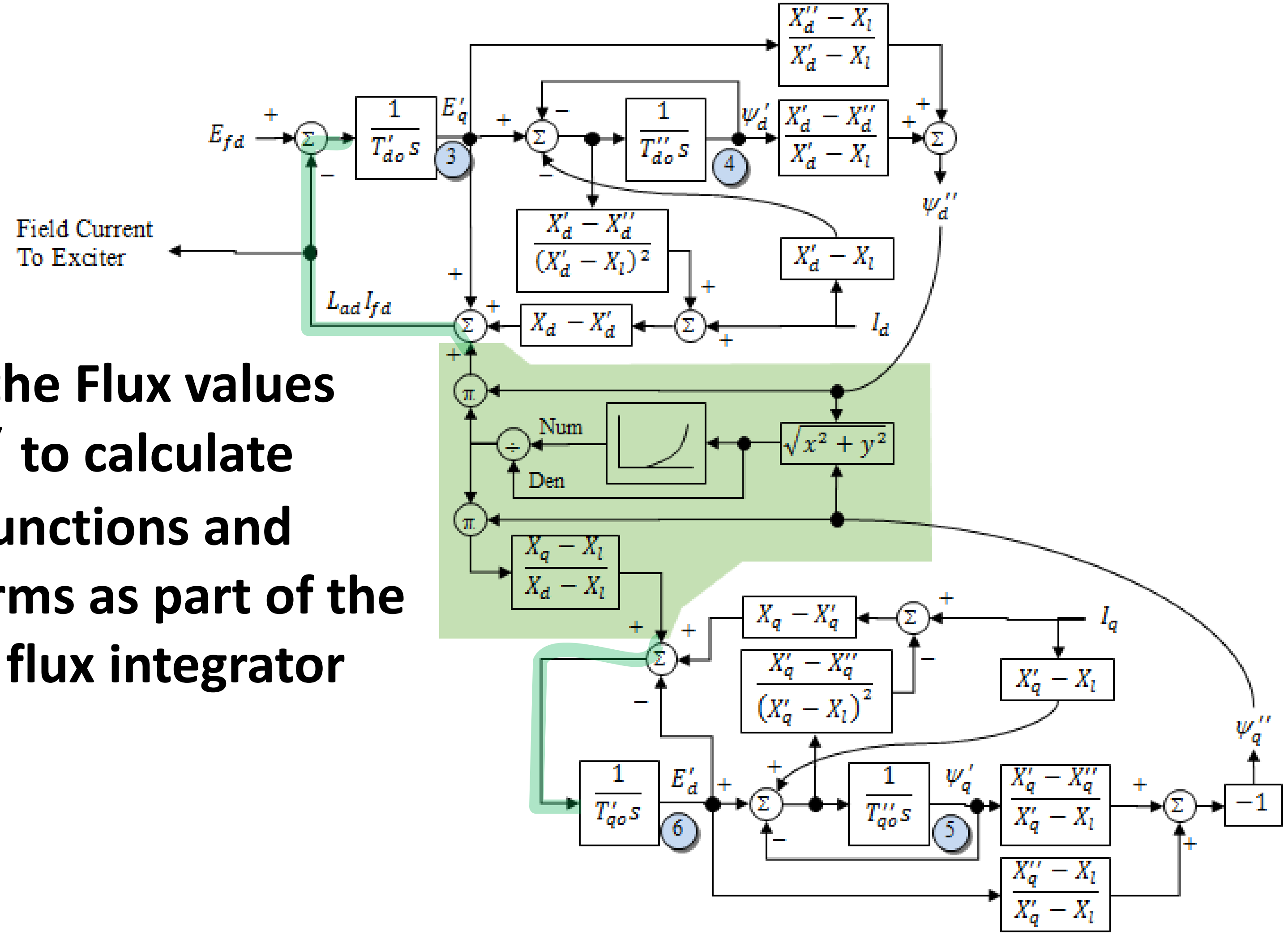
How do you model saturation?

- Ultimately saturation is always heuristic
- Based on fitting a function to measurements
- Different saturation functions are used

Name	Function	Which Platform
Quadratic	$Sat(x) = B(x-A)^2$	GE PSLF PowerWorld Simulator option
Scaled Quadratic	$Sat(x) = \frac{B(x-A)^2}{x}$	PTI PSS/E, PowerWorld Simulator Option
Exponential	$Sat(x) = Bx^A$	BPA IPF Specific models in PTI PSS/E Specific models in PowerWorld Simulator

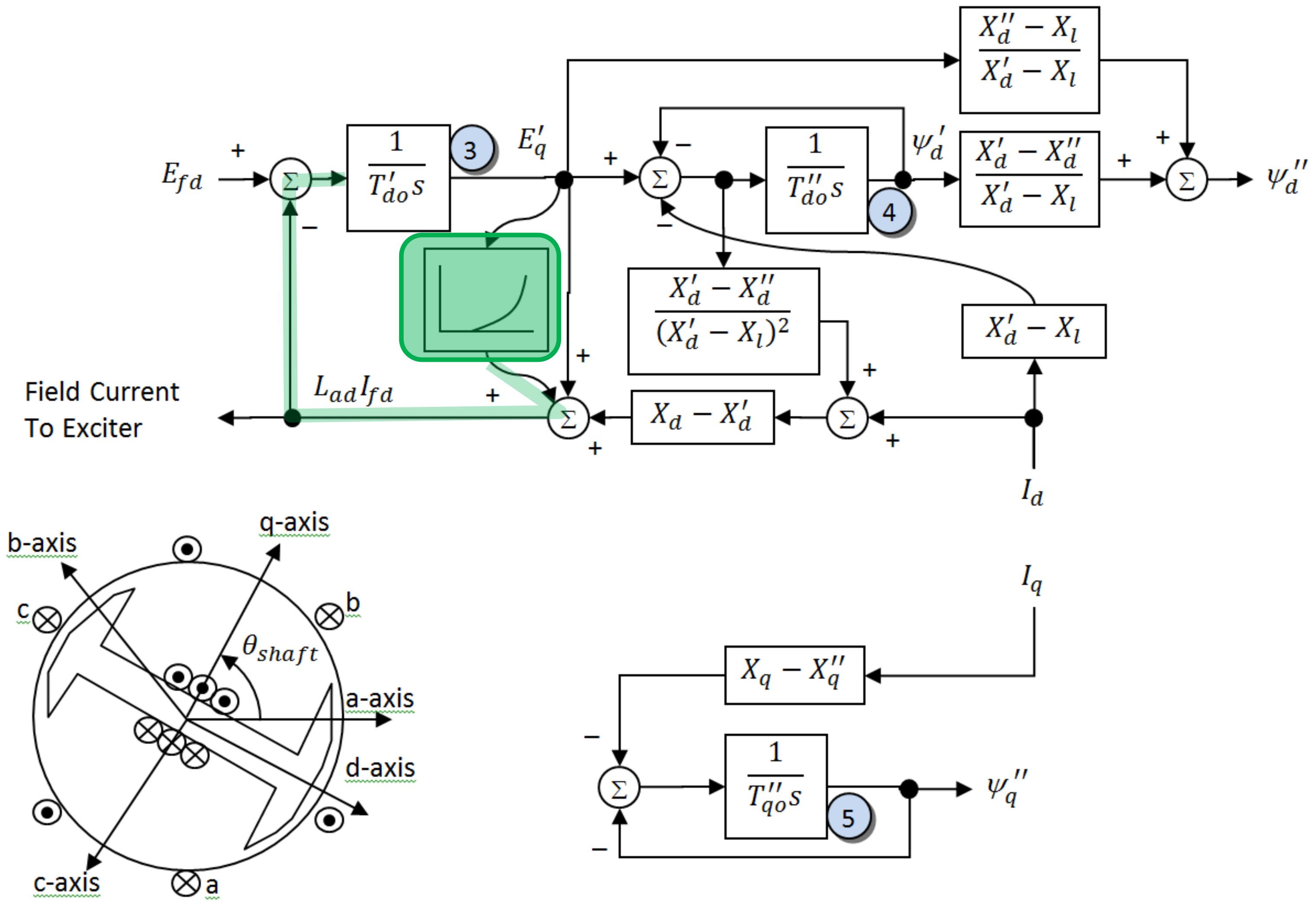
- How is saturation included in GENROU/GENSAL?
  - Terms are added to <u>*differential*</u> equations only
  - Network boundary equation is unchanged

Inclusion of Saturation as Additive terms



Terms take the Flux values ψ_q'' and ψ_d'' to calculate saturation functions and then add terms as part of the input to the flux integrator

GENSAL Model with Saturation



Implications of modeling saturation this way

- Delay in saturation affecting the results
  - Saturation in dynamic states will always have some delay because it only affects the input to an integrator
- Saturation does NOT impact the network boundary equations at all
  - As long as we require that $X''_d = X''_q$, a simple circuit equation can be used at network boundary
    - $X''_d <> X''_q$ is called transient saliency and is not allowed in GENROU and GENSAL models
  - This makes it much easier on software vendors and is likely a big reason why in 1970 this would have been picked as heuristic
- Saturation is only a function of the flux and thus the terminal voltage of the synchronous machine
  - Saturation is NOT a function of current

GENTPF and GENTPJ Model

- John Undrill has given us some references for this that are helpful
  - Motivation for need → Basically "it matches reality better" https://www.wecc.biz/Reliability/gentpj%20and%20gensal%20morel.pdf
  - Equations listed in 2nd page of document: https://www.wecc.biz/Reliability/gentpj-typej-definition.pdf

$$V_q = E_{q1} + E_{q2} - I_q R_a - I_d X_{ds} \quad (1)$$

$$V_d = E_{d1} + E_{d2} - I_d R_a + I_q X_{qs} \quad (2)$$

$$E"_q = E_{q1} + E_{q2} - I_d X_{ddds} \quad (3)$$

$$E"_d = E_{d1} + E_{d2} + I_q X_{qqqs} \quad (4)$$

$$E'_q = E_{q1} + E_{q2} - ((X'_d - X"_d)/(X_d - X"d))E_{q2} - I_d X_{dds} \quad (5)$$

$$E'_d = E_{d1} + E_{d2} - ((X'_q - X"_q)/(X_q - X"q))E_{d2} + I_q X_{qqs} \quad (6)$$

$$dE"_q/dt = -(1 + S_d)(((X'_d - X"_d)/(X_d - X"_d))E_{q2})/T"_{do} \quad (7)$$

$$dE"_d/dt = -(1 + S_q)(((X'_q - X"_q)/(X_q - X"_q))E_{d2})/T"_{qo} \quad (8)$$

$$dE'_q/dt = (E_{fd} - (1 + S_d)E_{q1})/T'_{do} \quad (9)$$

$$dE'_d/dt = -(1 + S_q)E_{d1}/T'_{qo} \quad (10)$$

$$X_{ds} = ((X_d - X_l)/(1 + S_d)) + X_l \quad (11)$$

$$X_{dds} = (X_d - X'_d)/(1 + S_d) \quad (12)$$

$$X_{ddds} = (X_d - X"d)/(1 + S_d) \quad (13)$$

$$X_{qs} = ((X_q - X_l)/(1 + S_q)) + X_l \quad (14)$$

$$X_{qqs} = (X_q - X'_q)/(1 + S_q) \quad (15)$$

$$X_{qqqs} = (X_q - X"q)/(1 + S_q) \quad (16)$$

$$E_l = sqrt((V_q + I_q R_a + I_d X_l)^2 + (V_d + I_d R_a - I_q X_l)^2) \quad (17)$$

$$I = sqrt(I_d^2 + I_q^2) \quad (18)$$

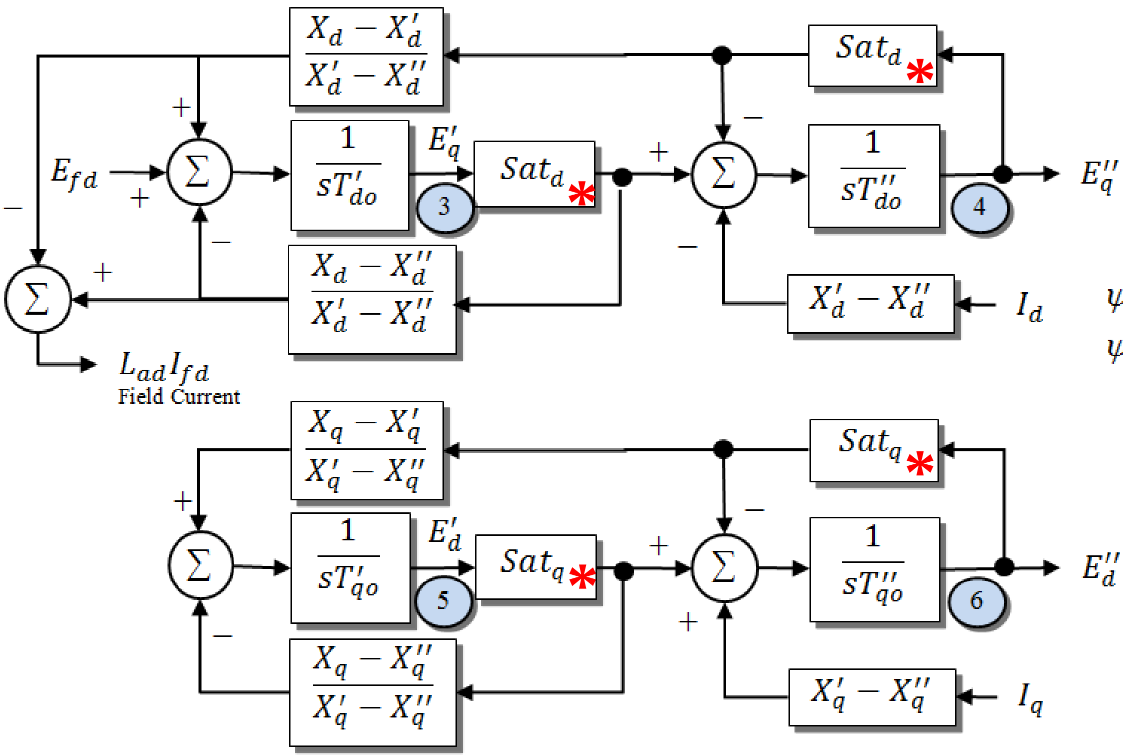
$$S_d = (saturation function)(E_l + K_{is} I) \quad (19)$$

$$S_q = (X_q/X_d)S_d \quad (20)$$

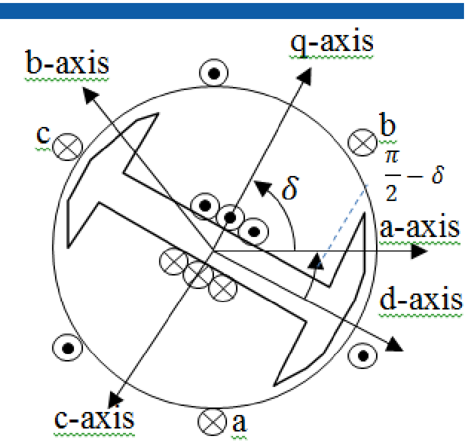
Comments on Undrill Writeup

- Refers to papers by D.W. Olive that serve as a foundation for GENPTF
  - Olive, D.W., “New Techniques for Calculation of Dynamic Stability”, IEEE Transactions on Power Apparatus and Systems, Vol PAS-85, No. 7, July 1966, pp. 767-777.
    - *Equations (9) – (14) in paper provide fundamental start of the GENTPF/GENTPJ equations.*
  - Olive, D.W., “Digital Simulation of Synchronous Machines Transients”, IEEE Transactions on Power Apparatus and Systems, Vol PAS-87, No. 8. Pp 1669-1675.
    - *Equations (18) – (27) in paper provide the extend earlier paper to more dynamic variables*
- I haven’t found a fundamental derivation of the D.W. Olive equations
  - Write up on PowerWorld’s website shows a full derivation showing that John’s equations ultimately result in the GENTPF model (almost), as well as how to derive GENTPF from GENROU equations http://www.powerworld.com/files/GENROU-GENSAL-GENTPF-GENTPJ.pdf

GENTPF/GENTPJ Differential Equations



$$\psi''_q = -E''_d$$
$$\psi''_d = +E''_q$$



$$\psi_{ag} = \sqrt{\left(V_{qterm} + I_q R_a + I_d X_l\right)^2 + \left(V_{dterm} + I_d R_a - I_q X_l\right)^2}$$

D-Axis: $Sat_d = 1 + Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Q-Axis: $Sat_q = 1 + \frac{X_q}{X_d} Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Saturation is implemented as *algebraic multiplication

- **All reactances saturate together**
- **Effects are instantaneous across entire model**

Network Boundary Equation Interface for GENTPF/GENTPJ

- $V_d + jV_q = (E''_d + jE''_d)(1+\omega)$
  - Internal voltage is the same as GENROU
- $X''_{dsat} = \dfrac{X''_d - X_l}{Sat_d} + X_l$ and $X''_{qsat} = \dfrac{X''_q - X_l}{Sat_q} + X_l$
- Network boundary equations are
    - $V_{dterm} = V_d - R_a I_d + X''_{qsat} I_q$
    - $V_{qterm} = V_q - X''_{dsat} I_d - R_a I_q$
  - Can't model network boundary equation as a circuit anymore because $X''_{qsat} <> X''_{dsat}$
    - $X''_q <> X''_d$ and $Sat_q <> Sat_d$.

Addition to John Undrill's Writeup

- Network boundary equations of GENTPF are slightly different than John Undrill's writeup.
  - John Undrill's writeup has the following
    - $V_{qterm} = E_{q1} + E_{q2} - I_d X_{ds} - I_q R_a$
    - $V_{dterm} = E_{d1} + E_{d2} + I_q X_{qs} - I_d R_a$
- Similar equations in D.W. Olive references include extra multiplication terms
    - $V_{qterm} = \left(E_{q1} + E_{q2} - I_d X_{ds}\right)(1 + \omega) - I_q R_a$
    - $V_{dterm} = \left(E_{d1} + E_{d2} + I_q X_{qs}\right)(1 + \omega) - I_d R_a$

Include the extra multiplication

- Extra term results in following network boundary equation
  - $V_{qterm} = +E''_q(1+\omega) - I_d X''_{dsat}(1+\omega) - I_q R_a$
  - $V_{dterm} = +E''_d(1+\omega) + I_q X''_{qsat}(1+\omega) - I_d R_a$

D.W Olive gives this

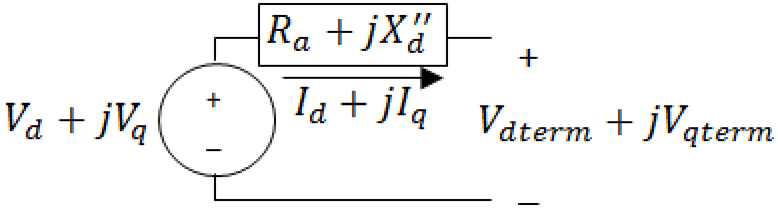
- This actually isn't what GENTPF/GENTPJ uses though!
  - Remove multiplication on network equation reactance
    - $V_{qterm} = +E''_q(1+\omega) - I_d X''_{dsat} - I_q R_a$
    - $V_{dterm} = +E''_d(1+\omega) + I_q X''_{qsat} - I_d R_a$

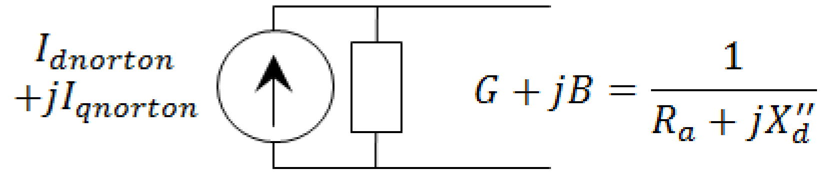
Software implements this

- Justification for removing multiplication
  - Modeling frequency impact on reactances in the network models (transmission lines and transformers) is not done anyway
    - Similar assumption made during that derivation of GENROU
    - All of these synchronous machine models are only valid near synchronous speed
  - This makes the GENTPF/GENTPJ much more like the GENROU/GENSAL which multiplies the flux terms by $(1+\omega)$, but not reactances

What if $X''_{dsat} = X''_{qsat}$?

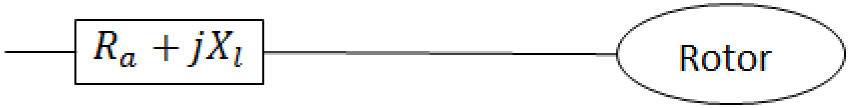
- GENTPF network equations become
  - $V_{qterm} = +E''_q(1+\omega) - I_d X''_{qsat} - I_q R_a$
  - $V_{dterm} = +E''_d(1+\omega) + I_q X''_{qsat} - I_d R_a$
- This can be written as a circuit again
  - $V_{dterm} + jV_{qterm}$
    $= \left(E''_d + jE''_q\right)(1+\omega) - \left(I_d + jI_q\right)\left(R_a + jX''_{qsat}\right)$
  - And we end up with same circuit as GENROU



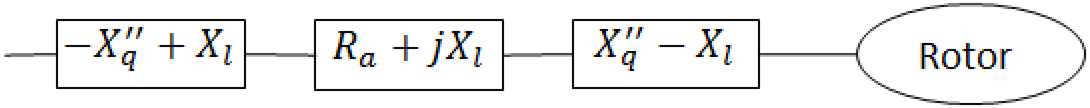


Theoretical Justification for GENTPJ/GENTPJ starting with GENROU

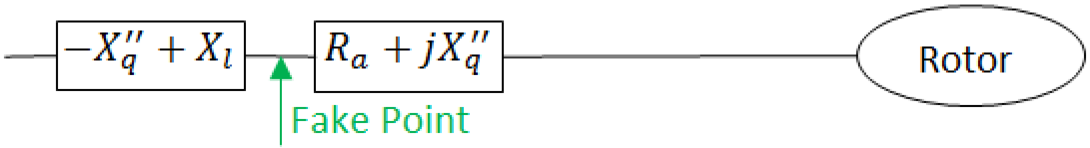
- Start with GENROU model without saturation
- GENROU includes armature resistance and reactance in series with each phase



- Add two additional impedances that sum to zero



- Then merge the second two terms



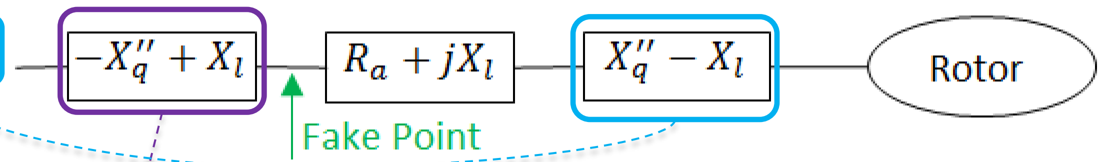
- Looking from the fake point inward, you just have GENROU with $X_l = X_q''$

Assume $X_l = X_q''$

- Details are on our website
  - Assume $X_l = X_q''$ with GENROU differential equations and you get …
    → GENTPF differential equations!
- However, we've modified the series impedance to the network
  - This impacts the network boundary equations
  - GENTPF/GENTPJ make network equations trickier

Effects on Network Boundary Equations

- Start by looking from the Fake Point.
- We added extra impedance which is subject to saturation, so take the impedance from GENROU derivation and add to it
  - $X''_{qfake} = X''_q + \frac{X''_q - X_l}{Sat_q}$
  - $X''_{dfake} = X''_d + \frac{X''_d - X_l}{Sat_d}$



- Then we added some impedance outside of the machine too so we need to add that
  - $X''_{qsat} = X''_q + \frac{X''_q - X_l}{Sat_q} - X''_q + X_l = \frac{X''_q - X_l}{Sat_q} + X_l = X''_{qsat}$
  - $X''_{dsat} = X''_d + \frac{X''_d - X_l}{Sat_d} - X''_d + X_l = \frac{X''_d - X_l}{Sat_d} + X_l = X''_{dsat}$

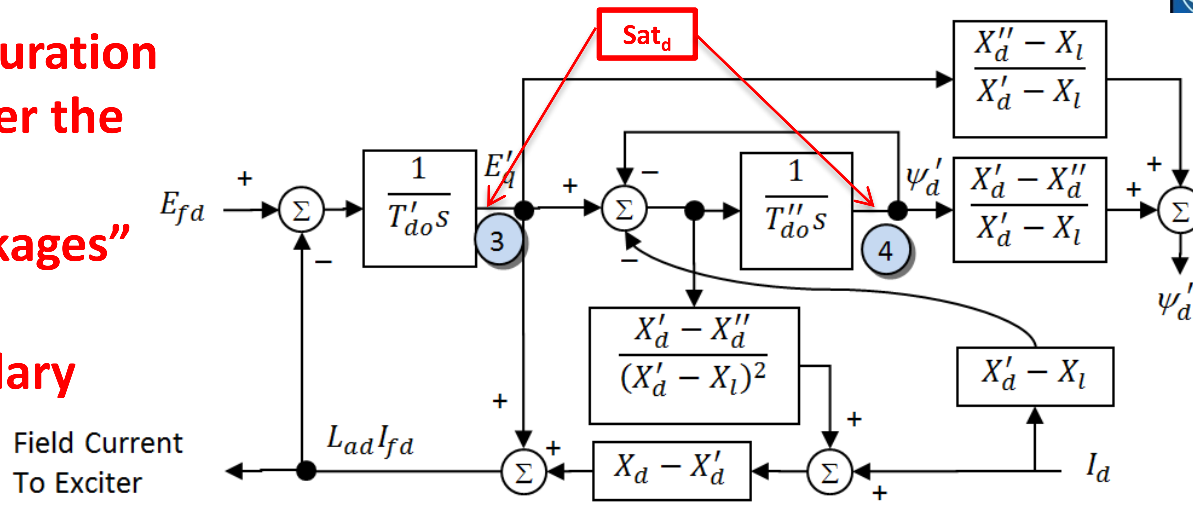
This is GENTPF

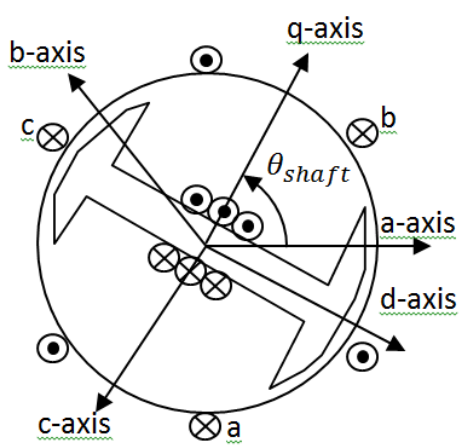
Implications of modeling GENTPF/GENTPJ model

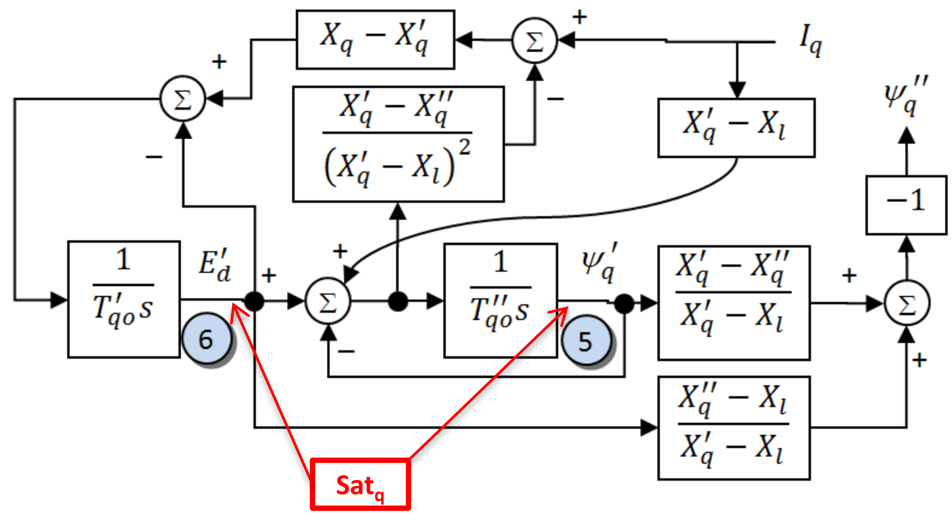
- Saturation effects are felt by all states of the model *immediately*
  - I suspect this is the biggest reason the model matches measurements better
- Saturation DOES impact the network boundary equation.
  - This means we can not use a simple circuit equation at the network boundary
- GENTPJ introduces saturation as a function of current.
  - $Sat_d = 1 + Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$
  - $Sat_q = 1 + \frac{X_q}{X_d} Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Another Possibility for GENTPF/GENTPJ

- **Add Multiplication by Saturation Function immediately after the integrator states**
- **States represent "flux linkages" so they saturate**
- **Use same network boundary equations as GENTPF/J**







Conclusion

- Write up is available on PowerWorld's website
  - http://www.powerworld.com/files/GENROU-GENSAL-GENTPF-GENTPJ.pdf
- Provides more theoretical treatment of a model that has been used for decades
- Transparency