Description of Machine Models GENROU, GENSAL, GENTPF and GENTPJ

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1   General Topics

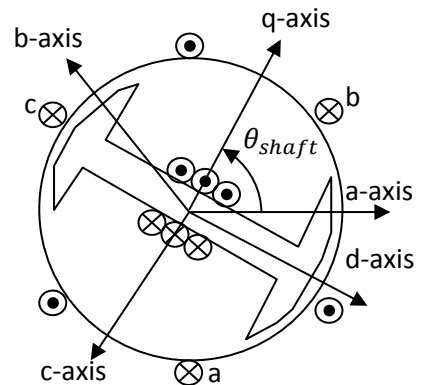## 1.1   Circuit Models for Network Interfaces (dq transformation)

It is very important to explicitly define the relationship between the network reference frame (abc phases) and the dq axis reference frame. When reading various textbooks you will see this done in at least 3 different ways. All references start by defining the d-axis using the right-hand rule with respect to the rotor windings. The phase axes (a,b,c) are then also defined using the right-hand rule with respect to the stator windings.
After these definitions though, there are two independent choices to be made.

1. Where is the q-axis? 90 degrees leading the d-axis or 90 degrees lagging
2. What is the rotor angle (θ_{shaft}) with respect to?

Software vendors have settled on the same choices as shown in the figure to the right.

1. q-axis is 90 degrees leading
2. Rotor angle is the angle of the q-axis with respect to the network a phase axis.

This matches the conventions used in the book *Power System Dynamics and Stability* by Peter Sauer and M.A. Pai. However you will find textbooks and other references that make different choices. As long as one is consistent in the choice it doesn't matter, however be very careful when comparing equations between textbooks and other references as they may use a different convention.

The following are circuit models for the network interface for the Thevenin and Norton Equivalents and on the dq axis and network reference frames.

Norton Equivalent			Thevenin Equivalent	
$I_{dnorton} + jI_{qnorton}$ ⊛	$G + jB = \dfrac{1}{R_a + jX_d''}$	dq Axis Reference Frame	$V_d + jV_q$ ⊛ $R_a + jX_d''$ $\overrightarrow{I_d + jI_q}$	$+$ $V_{dterm} + jV_{qterm}$ $-$
$I_{rnorton} + jI_{inorton}$ ⊛	$G + jB = \dfrac{1}{R_a + jX_d''}$	Network Reference Frame	$V_r + jV_i$ ⊛ $R_a + jX_d''$ $\overrightarrow{I_r + jI_i}$	$+$ $V_{rterm} + jV_{iterm}$ $-$

Conversions between reference frames are done using equation 6.8 and 6.9 on page 125 of Sauer/Pai book. The same general function works for converting all phasors between the dq axis and network reference frames. Assume you have a complex phase F that can be written as either $F_d + jF_q$ or $(F_r + jF_i)$.

$$F_d + jF_q = (F_r + jF_i)e^{-j\left(\delta-\frac{\pi}{2}\right)} = (F_r + jF_i)\left(\cos\left(\frac{\pi}{2} - \delta\right) + j\sin\left(\frac{\pi}{2} - \delta\right)\right) = (F_r + jF_i)\big(\sin(\delta) + j\cos(\delta)\big)$$

$$F_d = +F_r\sin(\delta) - F_i\cos(\delta)$$
$$F_q = +F_r\cos(\delta) + F_i\sin(\delta)$$

$$F_r + jF_i = (F_d + jF_q)e^{+j\left(\delta-\frac{\pi}{2}\right)} = (F_d + jF_q)\left(\cos\left(\delta - \frac{\pi}{2}\right) + j\sin\left(\delta - \frac{\pi}{2}\right)\right) = (F_d + jF_q)\big(\sin(\delta) - j\cos(\delta)\big)$$

$$F_r = +F_d\sin(\delta) + F_q\cos(\delta)$$
$$F_i = -F_d\cos(\delta) + F_q\sin(\delta)$$

1.2 Saturation Functions

There are three commonly used saturation functions used in power system transient stability software.

Name	Function	Which Platform
Quadratic	$Sat(x) = B(x - A)^2$	GE PSLF PowerWorld Simulator option
Scaled Quadratic	$Sat(x) = \dfrac{B(x - A)^2}{x}$	PTI PSS/E, PowerWorld Simulator Option
Exponential	$Sat(x) = Bx^A$	BPA IPF Specific models in PTI PSS/E Specific models in PowerWorld Simulator

Once a function is chosen, a model specifies the saturation function by giving two values of saturation function evaluated at 1.0 and 1.2 (call these values S_{10} and S_{12}. From these values you can then calculate the coefficents *A* and *B* of the respective equation.
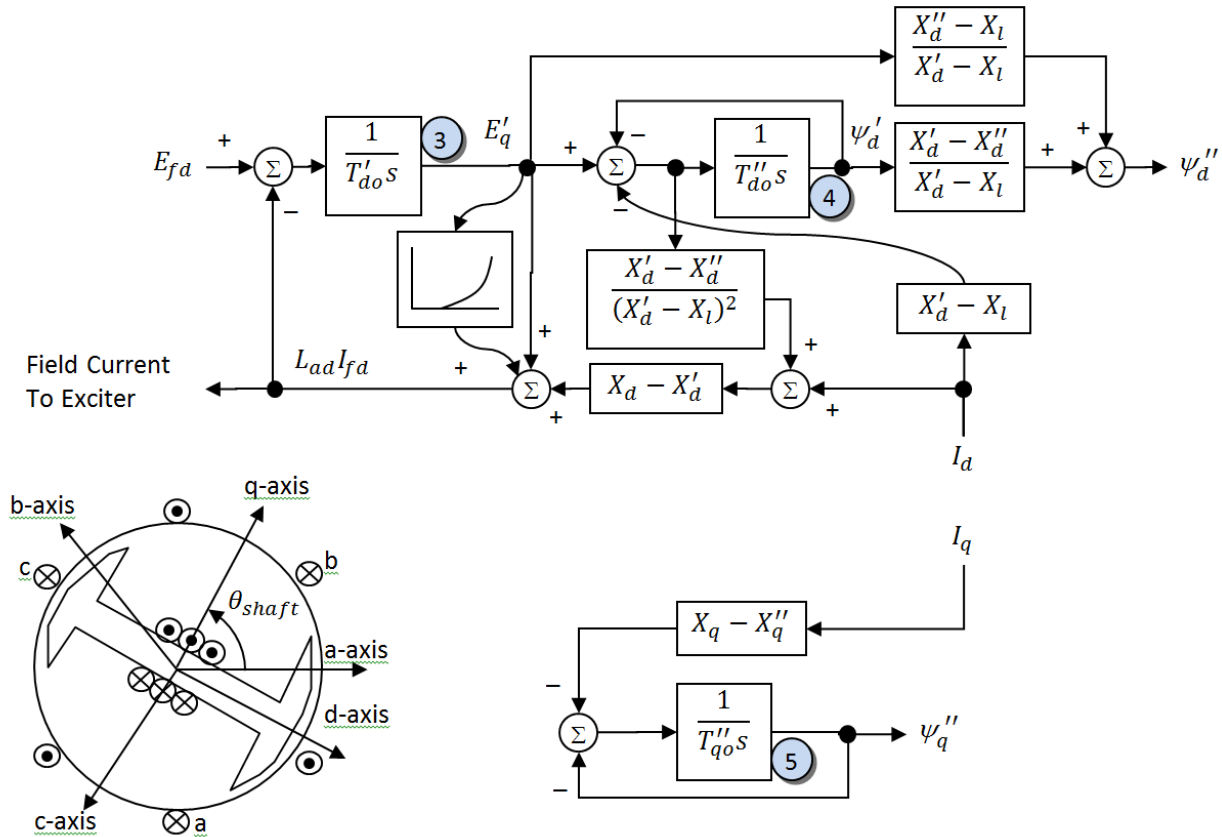
2 Machine Models with Additive Saturation

A fundamental derivation of a GENROU and GENSAL models can found in Chapter 3 of the book *Power System Dynamics and Stability* by Peter Sauer and M.A. Pai from 1998. This derivation starts from first principals represented by equations 3.1 – 3.9 on page 24 – 25 and culminates in Equations 3.148 – 3.159 on page 42. The equations on page 42 of Sauer/Pai essentially represent the block diagrams for GENROU and GENSAL without saturation (though there are some minor sign convention differences). The treatment of saturation is then discussed in more detail by including additive terms similar to what is done for GENROU and GENSAL. The treatment of saturation is one of the fundamental differences between GENROU/GENSAL and the GENTPF/GENTPJ models.

The following sub sections show the block diagrams and description of the network boundary interface for the following
- GENSAL model with saturation
- GENROU model without saturation
- GENROU model with saturation

This will then be followed by a section showing the block diagrams and description of the network boundary interface for the GENTPF and GENTPJ model along with some historical references to indicate where this model came from.

2.1   Machine: GENSAL model with saturation.

Electrical Torque

$$\psi_q = \psi_q'' - I_q X_d''$$

$$\psi_d = \psi_d'' - I_d X_d''$$

$$T_{elec} = \psi_d I_q - \psi_q I_d = \psi_d'' I_q - \psi_q'' I_d$$

Mechanical Swing Equations

① $\dot{\delta} = \omega * \omega_0$

② $\dot{\omega} = \frac{1}{2H}\left(\frac{P_{mech} - D\omega}{1+\omega} - T_{elec}\right)$

ω =   per unit speed deviation, so $\omega = 0$ means we are at synchronous speed and $\omega = 1$ would mean it's spinning at double synchronous speed

ω_0 =   synchronous speed $2\pi f_0$ where f_0 is the nominal system frequency in Hz

Note: If option *Ignore Speed Effects in Generator Swing Equation* is true, then instead use

$$\dot{\omega} = \frac{1}{2H}(P_{mech} - D\omega - T_{elec})$$

Network Interface Equations

$$Z_{source} = R_a + jX_d''$$

$$Y_{source} = \frac{1}{R_a + jX_d''} = G + jB$$

$$V_d + jV_q = \frac{d\left(\psi_d'' + j\psi_q''\right)}{dt}$$

$$V_d + jV_q = j(1+\omega)\left(\psi_d'' + j\psi_q''\right)$$

$$V_d + jV_q = \left(-\psi_q'' + j\psi_d''\right)(1+\omega)$$

$$I_{dnorton} + jI_{qnorton} = (V_d + jV_q)(G + jB)$$

Convert to Network Reference

$$I_{rnorton} + jI_{inorton} = \left(I_d + jI_q\right)e^{j(\delta - \frac{\pi}{2})}$$

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2.2   Machine: GENROU model without saturation

Electrical Torque

$$\psi_q = \psi_q'' - I_q X_d''$$

$$\psi_d = \psi_d'' - I_d X_d''$$

$$T_{elec} = \psi_d I_q - \psi_q I_d = \psi_d'' I_q - \psi_q'' I_d$$

Mechanical Swing Equations

(1) $\dot{\delta} = \omega * \omega_0$

(2) $\dot{\omega} = \frac{1}{2H}\left(\frac{P_{mech} - D\omega}{1+\omega} - T_{elec}\right)$

$\omega =$ per unit speed deviation, so $\omega = 0$ means we are at synchronous speed and $\omega = 1$ would mean it's spinning at double synchronous speed
$\omega_0 =$ synchronous speed $2\pi f_0$ where f_0 is the nominal system frequency in Hz
Note: If option *Ignore Speed Effects in Generator Swing Equation* is true, then instead use

$$\dot{\omega} = \frac{1}{2H}(P_{mech} - D\omega - T_{elec})$$

Network Interface Equations

$$Z_{source} = R_a + jX_d''$$

$$Y_{source} = \frac{1}{R_a + jX_d''} = G + jB$$
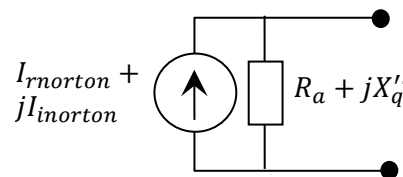
$$V_d + jV_q = \frac{d\left(\psi_d'' + j\psi_q''\right)}{dt}$$

$$V_d + jV_q = j(1+\omega)\left(\psi_d'' + j\psi_q''\right)$$

$$V_d + jV_q = \left(-\psi_q'' + j\psi_d''\right)(1+\omega)$$
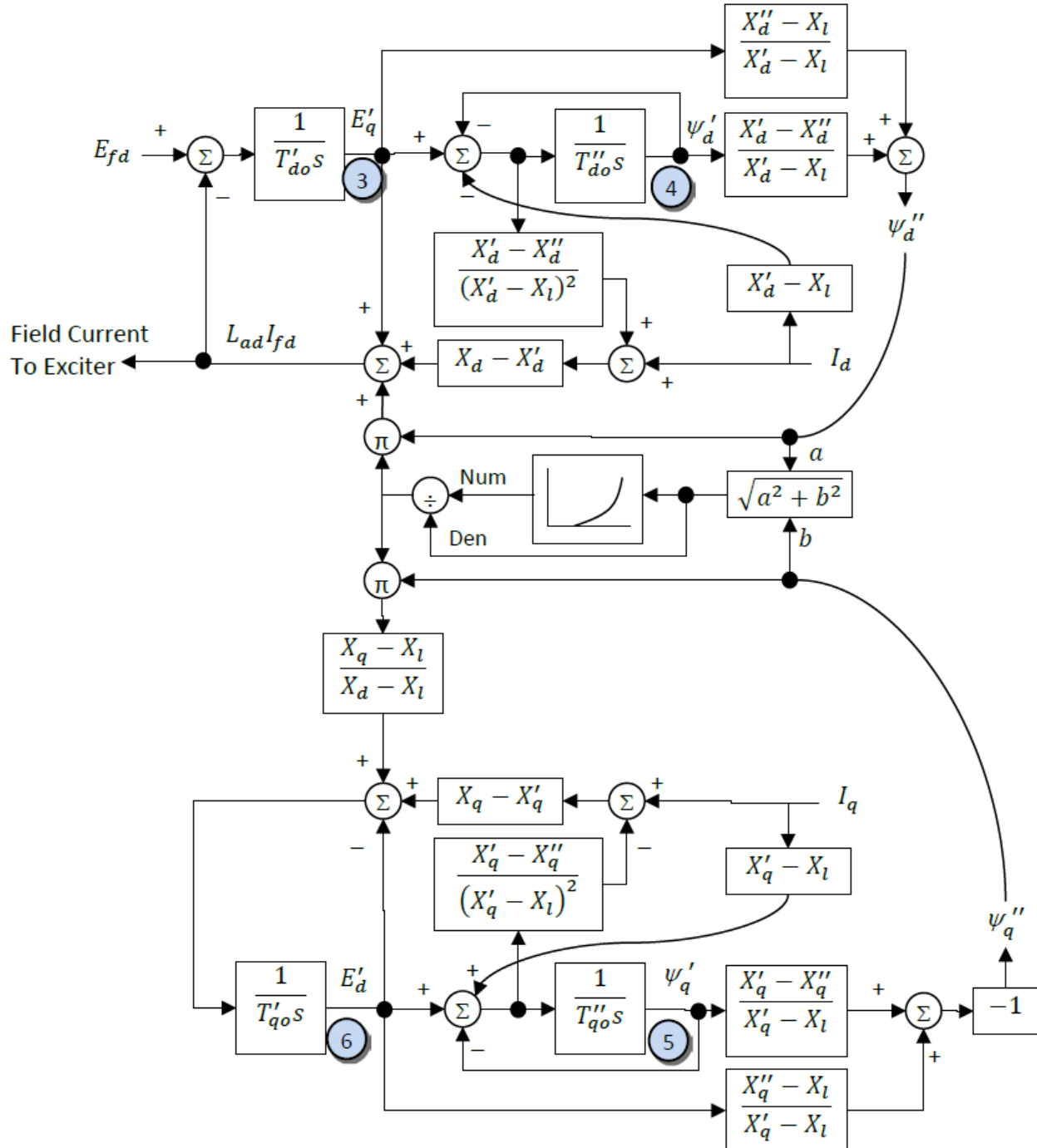
$$I_d + jI_q = (V_d + jV_q)(G + jB)$$

Convert to Network Reference

$$I_r + jI_i = \left(I_d + jI_q\right)e^{j\left(\delta - \frac{\pi}{2}\right)}$$
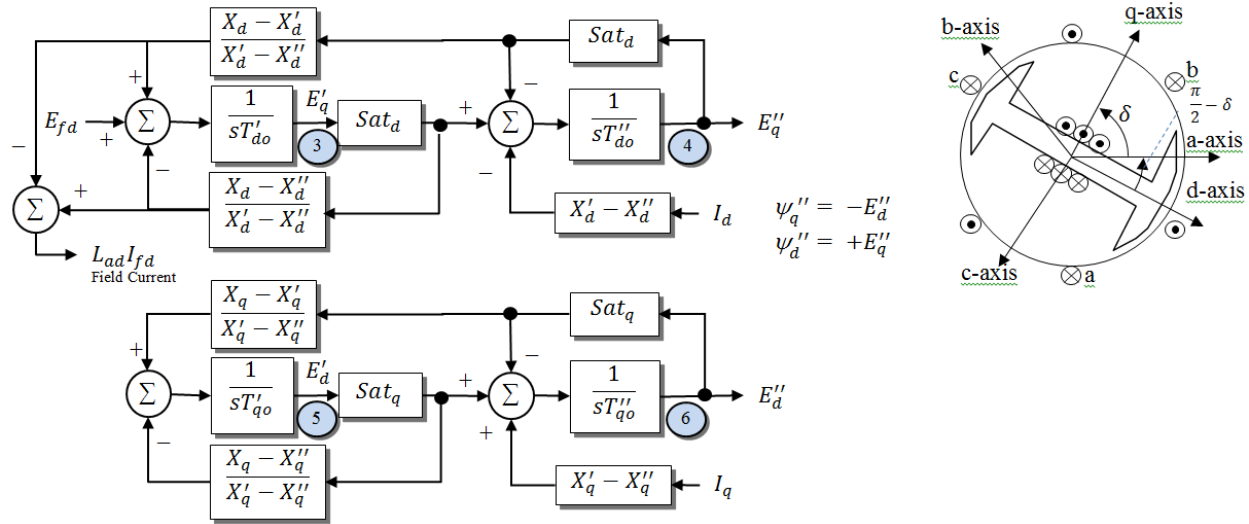
2.3 Machine: GENROU model with saturation

The GENROU model including saturation is presented below. The Mechanical Swing Equations, Electric Torque, and Network Interface Equations are all the same as the GENROU and GENSAL models previously.

$$E_{fd} \xrightarrow{+} \Sigma \xrightarrow{} \frac{1}{T'_{do}s} \;\; E'_q \;\; (3)$$

$$\frac{X''_d - X_l}{X'_d - X_l}$$

$$\frac{1}{T''_{do}s} \;\; \psi'_d \;\; (4) \quad \frac{X'_d - X''_d}{X'_d - X_l} \quad \Sigma \quad \psi''_d$$

$$\frac{X'_d - X''_d}{(X'_d - X_l)^2}$$

$$X'_d - X_l$$

Field Current To Exciter $\quad L_{ad}I_{fd} \quad \Sigma \quad X_d - X'_d \quad \Sigma \quad I_d$

$$\pi$$

$$a$$

$$\sqrt{a^2 + b^2}$$

Num Den $\div$

$$b$$

$$\pi$$

$$\frac{X_q - X_l}{X_d - X_l}$$

$$\Sigma \quad X_q - X'_q \quad \Sigma \quad I_q$$

$$\frac{X'_q - X''_q}{(X'_q - X_l)^2}$$

$$X'_q - X_l$$

$$\frac{1}{T'_{qo}s} \;\; E'_d \;\; (6) \quad \Sigma \quad \frac{1}{T''_{qo}s} \;\; \psi'_q \;\; (5) \quad \frac{X'_q - X''_q}{X'_q - X_l} \quad \Sigma \quad -1 \quad \psi''_q$$

$$\frac{X''_q - X_l}{X'_q - X_l}$$

3 History of Machine: GENTPF and GENTPJ

$$\psi_{ag} = \sqrt{\left(V_{qterm} + I_q R_a + I_d X_l\right)^2 + \left(V_{dterm} + I_d R_a - I_q X_l\right)^2}$$

D-Axis: $Sat_d = 1 + Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Q-Axis: $Sat_q = 1 + \dfrac{X_q}{X_d} Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Saturated Subtransient Reactances

$$X''_{dsat} = \frac{X''_d - X_l}{Sat_d} + X_l$$

$$X''_{qsat} = \frac{X''_q - X_l}{Sat_q} + X_l$$

Note: $X''_{qsat} <> X''_{dsat}$ both because $X''_q <> X''_d$ and because $Sat_q <> Sat_d$.

Mechanical Swing Equations

① $\dot\delta = \omega * \omega_0$

② $\dot\omega = \dfrac{1}{2H}\left(\dfrac{P_{mech} - D\omega}{1 + \omega} - T_{elec}\right)$

$\omega =$ per unit speed deviation, so $\omega = 0$ means we are at synchronous speed and $\omega = 1$ would mean it's spinning at double synchronous speed

$\omega_0 =$ synchronous speed $2\pi f_0$ where f_0 is the nominal system frequency in Hz

Note: If option *Ignore Speed Effects in Generator Swing Equation* is true, then instead use

$$\dot\omega = \frac{1}{2H}\left(P_{mech} - D\omega - T_{elec}\right)$$

Electrical Torque

Corrections made to dq subscripts of the X terms on December 3, 2015

$$\psi_q = \psi''_q - I_q X''_{qsat} = -E''_d - I_q X''_{qsat}$$

$$\psi_d = \psi''_d - I_d X''_{dsat} = E''_q - I_d X''_{dsat}$$

$$T_{elec} = \psi_d I_q - \psi_q I_d$$

Network Interface Equations

$$V_d + jV_q = \frac{d\left(\psi''_d + j\psi''_q\right)}{dt}$$

$$V_d + jV_q = j(1 + \omega)\left(\psi''_d + j\psi''_q\right)$$

$$V_d + jV_q = \left(-\psi''_q + j\psi''_d\right)(1 + \omega)$$

Corrections made to E''_q subscript on December 3, 2015

$$V_d + jV_q = \left(E''_d + jE''_q\right)(1 + \omega)$$

Because $X''_{qsat} <> X''_{dsat}$ we cannot use a circuit model for network interface equations and must instead directly use the following.

$$V_{dterm} = V_d - R_a I_d + X''_{qsat} I_q$$

$$V_{qterm} = V_q - X''_{dsat} I_d - R_a I_q$$

Convert to Network Reference

$$I_r + jI_i = \left(I_d + jI_q\right)e^{j\left(\delta - \frac{\pi}{2}\right)}$$

$$V_r + jV_i = \left(V_d + jV_q\right)e^{j\left(\delta - \frac{\pi}{2}\right)}$$

A fundamental derivation of a GENROU and GENSAL models can found in Chapter 3 of the book *Power System Dynamics and Stability* by Peter Sauer and M.A. Pai from 1998. This derivation starts from first principals represented by equations 3.1 – 3.9 on page 24 – 25 and culminates in Equations 3.148 – 3.159 on page 42. The equations on page 42 of Sauer/Pai essentially represent the block diagrams for GENROU and GENSAL without saturation (though there are some minor sign convention differences). Saturation is then discussed in more detail by including additive terms similar to what is done for GENROU and GENSAL. We have not found a fundamental derivation of the GENTPF/GENTPJ model equations. There are papers from the late 1960s which show equations very similar to them but the papers present them without the fundamental derivation as found in Sauer/Pai. These papers are

- Olive, D.W., "New Techniques for Calculation of Dynamic Stability", IEEE Transactions on Power Apparatus and Systems, Vol PAS-85, No. 7, July 1966, pp. 767-777.
  Equations (9) – (14) in this paper provide the fundamental start of the GENTPF/GENTPJ equations.
- Olive, D.W., "Digital Simulation of Synchronous Machines Transients", IEEE Transactions on Power Apparatus and Systems, Vol PAS-87, No. 8. Pp 1669-1675.
  Equations (18) – (27) in this paper provide the extend the earlier paper to more dynamic variables

The full equations used for GENTPF/GENTPJ were consolidated in a very short 2 page write-up by John Undrill for WECC on November 19, 2007 and then corrected on June 19, 2012. A link to this paper as of October 22, 2015 is found at https://www.wecc.biz/Reliability/gentpj-typej-definition.pdf .

In this section we will derive how the equations given in John Undrill's writeup result in the block diagram and network interface algebra used in the GENTPF/GENTPJ models. We will also then provide the theoretical framework showing that the GENROU and GENTPF/GENTPJ equations are the same but with saturation modeled differently.

3.1 Equations for GENTPF/GENTPJ to Block Diagram

The following are the equations given in the write-up from John Undrill for WECC in June 19, 2012, we have replaced the use of $(1 + S_d)$ and $(1 + S_q)$ with purely (Sat_d) and (Sat_q) and also renamed the term V_{qterm} and V_{dterm} which were written as V_q and V_d in the original Undrill document. We have made these changes to be consistent with the remainder of this document.

1. $V_{qterm} = E_{q1} + E_{q2} - I_d X_{ds} - I_q R_a$
2. $V_{dterm} = E_{d1} + E_{d2} + I_q X_{qs} - I_d R_a$
3. $E_q'' = E_{q1} + E_{q2} - I_d X_{ddds}$
4. $E_d'' = E_{d1} + E_{d2} + I_q X_{qqqs}$
5. $E_q' = E_{q1} + E_{q2} - \frac{(X_d' - X_d'')}{(X_d - X_d'')} E_{q2} - I_d X_{dds}$
6. $E_d' = E_{d1} + E_{d2} - \frac{(X_q' - X_q'')}{(X_q - X_q'')} E_{d2} + I_q X_{qqs}$
7. $\frac{dE_q''}{dt} = -(Sat_d) \frac{(X_d' - X_d'')}{(X_d - X_d'')} \frac{E_{q2}}{T_{do}''}$
8. $\frac{dE_d''}{dt} = -(Sat_q) \frac{(X_q' - X_q'')}{(X_q - X_q'')} \frac{E_{d2}}{T_{qo}''}$
9. $\frac{dE_q'}{dt} = \frac{E_{fd} - (Sat_d) E_{q1}}{T_{do}'}$
10. $\frac{dE_d'}{dt} = \frac{-(Sat_q) E_{d1}}{T_{qo}'}$

11. $X_{ds} = \frac{(X_d - X_l)}{Sat_d} + X_l$

12. $X_{dds} = \frac{(X_d - X_d')}{Sat_d}$

13. $X_{ddds} = \frac{(X_d - X_d'')}{Sat_d}$

14. $X_{qs} = \frac{(X_q - X_l)}{Sat_q} + X_l$

15. $X_{qqs} = \frac{(X_q - X_q')}{Sat_q}$

16. $X_{qqqs} = \frac{(X_q - X_q'')}{Sat_q}$

17. $\psi_{ag} = \sqrt{\left(V_{qterm} + I_q R_a + I_d X_l\right)^2 + \left(V_{dterm} + I_d R_a - I_q X_l\right)^2}$

18. $Sat_d = 1 + Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

19. $Sat_q = 1 + \frac{X_q}{X_d} Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

While it is not immediately clear that this list of equation becomes the GENTPF/GENTPJ model shown in the block diagram. The following derivation will show this.

Take Equation 13 and 16 add substitute back into equation 3 and 4.

$$E_{q1} + E_{q2} = +I_d \frac{(X_d - X_d'')}{Sat_d} + E_q''$$

$$E_{d1} + E_{d2} = -I_q \frac{(X_q - X_q'')}{Sat_q} + E_d''$$

Substitute these equations along with 12 and 14 back into equation 5 and 6.

$$E_q' = +I_d \frac{(X_d - X_d'')}{Sat_d} + E_q'' - \frac{(X_d' - X_d'')}{(X_d - X_d'')} E_{q2} - I_d \frac{(X_d - X_d')}{Sat_d}$$

$$E_d' = -I_q \frac{(X_q - X_q'')}{Sat_q} + E_d'' - \frac{(X_q' - X_q'')}{(X_q - X_q'')} E_{d2} + I_q \frac{(X_q - X_q')}{Sat_q}$$

Move terms to the left side

$$E_q' - E_q'' - I_d \frac{(X_d - X_d'')}{Sat_d} + I_d \frac{(X_d - X_d')}{Sat_d} = -\frac{(X_d' - X_d'')}{(X_d - X_d'')} E_{q2}$$

$$E_d' - E_d'' + I_q \frac{(X_q - X_q'')}{Sat_q} - I_q \frac{(X_q - X_q')}{Sat_q} = -\frac{(X_q' - X_q'')}{(X_q - X_q'')} E_{d2}$$

Multiply by Sat_d and Sat_q on appropriate terms and group terms multiplied by I_d and I_q.

$$(Sat_d)(E_q' - E_q'') - I_d(X_d' - X_d'') = -(Sat_d)\frac{(X_d' - X_d'')}{(X_d - X_d'')} E_{q2}$$

$$(Sat_q)(E_d' - E_d'') + I_q(X_q' - X_q'') = -(Sat_q)\frac{(X_q' - X_q'')}{(X_q - X_q'')} E_{d2}$$

The right-hand side of these two equations match the right-hand side of equations 7 and 8. Therefore

$$\frac{dE_q''}{dt} = \frac{(Sat_d)(E_q' - E_q'') - I_d(X_d' - X_d'')}{T_{do}''}$$

$$\frac{dE_d''}{dt} = \frac{(Sat_q)(E_d' - E_d'') + I_q(X_q' - X_q'')}{T_{qo}''}$$

Now take same equation and solve for E_{q2} and E_{d2}.

$$-E_{q2} = \left(\left(E_q' - E_q''\right) - I_d \frac{(X_d' - X_d'')}{(Sat_d)}\right) \frac{(X_d - X_d'')}{(X_d' - X_d'')} = \left(E_q' - E_q''\right)\frac{(X_d - X_d'')}{(X_d' - X_d'')} - I_d \frac{(X_d - X_d'')}{(Sat_d)}$$

$$-E_{d2} = \left((E_d' - E_d'') + I_q \frac{X_q' - X_q''}{(Sat_q)} \right) \frac{(X_q - X_q'')}{(X_q' - X_q'')} = (E_d' - E_d'') \frac{(X_q - X_q'')}{(X_q' - X_q'')} + I_q \frac{(X_q - X_q'')}{(Sat_q)}$$

Now let's go back to equation 13 and 16 substituted into equation 3 and 4 and then move the terms E_{q2} and E_{d2} to the right-hand side.

$$E_{q1} = +I_d \frac{(X_d - X_d'')}{Sat_d} + E_q'' - E_{q2}$$

$$E_{d1} = -I_q \frac{(X_q - X_q'')}{Sat_q} + E_d'' - E_{d2}$$

Take our solution for E_{q2} and E_{d2} and substitute those in.

$$E_{q1} = +I_d \frac{(X_d - X_d'')}{Sat_d} + E_q'' + (E_q' - E_q'') \frac{(X_d - X_d'')}{(X_d' - X_d'')} - I_d \frac{(X_d - X_d'')}{(Sat_d)}$$

$$E_{d1} = -I_q \frac{(X_q - X_q'')}{Sat_q} + E_d'' + (E_d' - E_d'') \frac{(X_q - X_q'')}{(X_q' - X_q'')} + I_q \frac{(X_q - X_q'')}{(Sat_q)}$$

Note that the terms multiplied by I_d and I_q both cancel.

$$E_{q1} = +E_q'' + (E_q' - E_q'') \frac{(X_d - X_d'')}{(X_d' - X_d'')}$$

$$E_{d1} = +E_d'' + (E_d' - E_d'') \frac{(X_q - X_q'')}{(X_q' - X_q'')}$$

Then simplify further by grouping terms

$$E_{q1} = +E_q'' \left(1 - \frac{(X_d - X_d'')}{(X_d' - X_d'')} \right) + E_q' \frac{(X_d - X_d'')}{(X_d' - X_d'')} = -E_q'' \left(\frac{X_d - X_d'}{X_d' - X_d''} \right) + E_q' \left(\frac{X_d - X_d''}{X_d' - X_d''} \right)$$

$$E_{d1} = +E_d'' \left(1 - \frac{(X_q - X_q'')}{(X_q' - X_q'')} \right) + E_d' \frac{(X_q - X_q'')}{(X_q' - X_q'')} = -E_d'' \left(\frac{X_q - X_q'}{X_q' - X_q''} \right) + E_d' \left(\frac{X_q - X_q''}{X_q' - X_q''} \right)$$

Now substitute these equations into equations 9 and 10.

$$\frac{dE_q'}{dt} = \frac{E_{fd} + (Sat_d) \left(E_q'' \left(\frac{X_d - X_d'}{X_d' - X_d''} \right) - E_q' \left(\frac{X_d - X_d''}{X_d' - X_d''} \right) \right)}{T_{do}'}$$

$$\frac{dE_d'}{dt} = \frac{+(Sat_q) \left(E_d'' \left(\frac{X_q - X_q'}{X_q' - X_q''} \right) - E_d' \left(\frac{X_q - X_q''}{X_q' - X_q''} \right) \right)}{T_{qo}'}$$

If you look the block diagram for GENTPF/GENTPJ you will find that the equations for $\frac{dE_q'}{dt}, \frac{dE_d'}{dt}, \frac{dE_q''}{dt}$, and $\frac{dE_d''}{dt}$ all match these equations.

Now let's see how we obtain the network interface equations used in GENTPF/GENTPJ from John Undrill's equations. As we started the dynamic equation derivation, take Equation 13 and 16 add substitute back into equation 3 and 4 and then substitute the values back into Equation 1 and 2. Also take equation 11 and 14 to substitute in for X_{ds} and X_{qs}.

$$V_{qterm} = +E_q'' + I_d \frac{(X_d - X_d'')}{Sat_d} - I_d \left(\frac{(X_d - X_l)}{Sat_d} + X_l \right) - I_q R_a$$

$$V_{dterm} = +E_d'' - I_q \frac{(X_q - X_q'')}{Sat_q} + I_q \left(\frac{(X_q - X_l)}{Sat_q} + X_l \right) - I_d R_a$$

Now group terms to simplify.

$$V_{qterm} = +E_q'' - I_d \left(\frac{(X_d'' - X_l)}{Sat_d} + X_l \right) - I_q R_a$$

$$V_{dterm} = +E_d'' + I_q \left(\frac{(X_q'' - X_l)}{Sat_q} + X_l \right) - I_d R_a$$

If we now define the terms

$$X_{dsat}'' = \frac{X_d'' - X_l}{Sat_d} + X_l$$

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$$X''_{qsat} = \frac{X''_q - X_l}{Sat_q} + X_l$$

Then we may write

$$V_{qterm} = +E''_q - I_d X''_{dsat} - I_q R_a$$
$$V_{dterm} = +E''_d + I_q X''_{qsat} - I_d R_a$$

These almost match our equations from the block diagram description. There is one additional term missing from the GENTPF and GENTPJ block diagrams. There is an additional multiplication by per unit speed on the E''_q and E''_d term. Thus our actual implementation of the GENTPF and GENTPJ models is as follows.

$$V_{qterm} = +E''_q(1 + \omega) - I_d X''_{dsat} - I_q R_a$$
$$V_{dterm} = +E''_d(1 + \omega) + I_q X''_{qsat} - I_d R_a$$

This is partially consistent with the original papers from D.W. Olive. On page 1671 of the original D.W. Olive paper from 1968 (which is also referenced in John Undrill's writeup for WECC), there are equations 18 and 19 which would correspond to a minor change to Equation 1 and 2 of Undrill's writeup for WECC as show by multiply the flux and reactance terms by per unit speed $(1 + \omega)$.

$$V_{qterm} = \left(E_{q1} + E_{q2} - I_d X_{ds}\right)(1 + \omega) - I_q R_a$$
$$V_{dterm} = \left(E_{d1} + E_{d2} + I_q X_{qs}\right)(1 + \omega) - I_d R_a$$

Including this extra multiplication by per unit speed would result in the following network interface equation.

$$V_{qterm} = +E''_q(1 + \omega) - I_d X''_{dsat}(1 + \omega) - I_q R_a$$
$$V_{dterm} = +E''_d(1 + \omega) + I_q X''_{qsat}(1 + \omega) - I_d R_a$$

Thus the implementation in the software tools ignore the impact of speed variation on the term associated with reactance, but continue to include the impact of speed on the terms associated with flux linkage. This is in keeping with the history of the GENSAL and GENROU models which also multiply their flux linkage value by per unit speed. It is also in keeping with the choice in transient stability models to not include the impact of frequency changes on the reactance values in the transmission system.

3.2 Theoretical Justification for GENTPF/GENTPJ

As a starting point, this section will take the fundamental derivation of a GENROU and GENSAL models found in Chapter 3 of the book *Power System Dynamics and Stability* by Peter Sauer and M.A. Pai from 1986. We will then show how to convert this model into GENTPF/GENTPJ. Essentially all of this revolves around how best to model saturation in the synchronous machine.

In the GENROU/GENSAL models the saturation shows up purely as an additive term the derivative of E'_q and E'_d. In GENROU/GENSAL models, the saturation does not come into play in the equations for the network interface at all and thus when the assumption that $X''_q = X''_d$ is made in GENROU/GENSAL the network equations can be modeled as a simple circuit model. (Note: when $X''_q <> X''_d$, this is called transient saliency.)

The advantage of the GENTPF and GENTPJ model is that the saturation really affects the entire model as the saturation is applied directly to all the inductance terms simultaneously. This likely better reflects what occurs due to the actual physics of the device. This does however complicate the network boundary equations as the saturation will be included in the network interface equations and thus a simple circuit model can not be used. However, because the GENTPF/GENTPJ model is from the start designed to permit transient saliency with $X''_q <> X''_d$ anyway, there is no additional increase in complexity by having saturation affect the network equations.
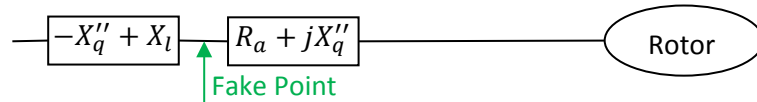
To convert the GENROU equations into the GENTPF/GENTPJ equations do the follow. The rotor model includes an armature resistance and leakage reactance in series with each phase as shown in the following picture.
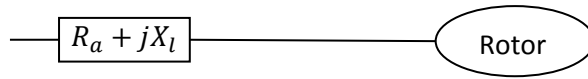
Consider adding two additional impedances in series which cancel one another out.
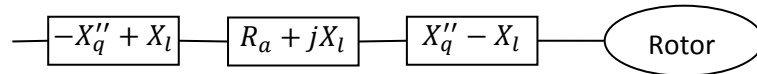
Then merge the second two terms simplify this to

Now looking from the "Fake Point" inward, this will look like a GENROU/GENSAL model but with $X_l = X_q''$ and also some "extra" Norton impedance of $X_q'' - X_l$ for the network interface.

We then need to know what the resulting Norton impedance for use in the network equations look like from this *fake point*. In the derivation from Sauer/Pai the Norton reactances would be X_q'' and X_d'', but we have added in some extra impedance $X_q'' - X_l$. This extra impedance however will become saturated so the total impedance seen from the *fake point* would be as follows.

$$X_{qfake}'' = X_q'' + \frac{X_q'' - X_l}{Sat_q}$$

$$X_{dfake}'' = X_d'' + \frac{X_d'' - X_l}{Sat_d}$$

We then need to add in the additional impedance in front of the *fake point* so we get the total Norton impedance of

$$X_{qsat}'' = X_q'' + \frac{X_q'' - X_l}{Sat_q} - X_q'' + X_l = \frac{X_q'' - X_l}{Sat_q} + X_l$$

$$X_{dsat}'' = X_d'' + \frac{X_d'' - X_l}{Sat_d} - X_d'' + X_l = \frac{X_d'' - X_l}{Sat_d} + X_l$$

These impedances match those given in the GENTPF/GENTPJ model.

We must then show how the GENROU derivative equations change into the GENTPF/GENTPJ equations. From the block diagram for GENROU

$$+E_q'' = \psi_d'' = +E_q' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_d' \frac{X_d' - X_d''}{X_d' - X_l} = +\psi_d'$$

$$-E_d'' = \psi_q'' = -E_d' \frac{X_q'' - X_l}{X_q' - X_l} - \psi_q' \frac{X_q' - X_q''}{X_q' - X_l} = -\psi_q'$$

Thus $E_q'' = \psi_d'$ and $E_d'' = \psi_q'$. Substitute that into the derivate equations and you get the following.

Conversion for $\frac{dE'_q}{dt}$. Start with GENROU equation and substitute $E''_q = \psi'_d$ and $X_l = X''_q$

$$\frac{dE'_q}{dt} = \frac{E_{fd}-\left(E'_q+(X_d-X'_d)\left(I_d+\left(\frac{x'_d-x''_d}{\left(x'_d-x_l\right)^2}\right)\left(-\psi'_d-(X'_d-X_l)I_d+E'_q\right)\right)\right)}{T'_{do}}$$

$$\frac{dE'_q}{dt} = \frac{E_{fd}-\left(E'_q+(X_d-X'_d)\left(I_d+\left(\frac{x'_d-x''_d}{\left(x'_d-x''_d\right)^2}\right)\left(-E''_q-(X'_d-X''_d)I_d+E'_q\right)\right)\right)}{T'_{do}}$$

$$\frac{dE'_q}{dt} = \frac{E_{fd}-\left(E'_q+(X_d-X'_d)\left(I_d-\frac{E''_q}{x'_d-x''_d}-I_d+\frac{E'_q}{x'_d-x''_d}\right)\right)}{T'_{do}}$$

$$\frac{dE'_q}{dt} = \frac{E_{fd}-\left(E'_q+\left(\frac{x_d-x'_d}{x'_d-x''_d}\right)E'_q-\left(\frac{x_d-x'_d}{x'_d-x''_d}\right)E''_q\right)}{T'_{do}}$$

$$\frac{dE'_q}{dt} = \frac{E_{fd}-\left(\frac{x_d-x''_d}{x'_d-x''_d}\right)E'_q+\left(\frac{x_d-x'_d}{x'_d-x''_d}\right)E''_q}{T'_{do}}$$

Conversion for $\frac{dE'_d}{dt}$. Start with GENROU equation and substitute $E''_d = \psi'_q$ and $X_l = X''_q$.

$$\frac{dE'_d}{dt} = \frac{-E'_d+(X_q-X'_q)\left(I_q-\frac{x'_q-x''_q}{\left(x'_q-x_l\right)^2}(-\psi'_q+(X'_q-X_l)I_q+E'_d)\right)}{T'_{qo}}$$

$$\frac{dE'_d}{dt} = \frac{-E'_d+(X_q-X'_q)\left(I_q-\frac{x'_q-x''_q}{\left(x'_q-x''_q\right)^2}(-E''_d+(X'_q-X''_q)I_q+E'_d)\right)}{T'_{qo}}$$

$$\frac{dE'_d}{dt} = \frac{-E'_d+(X_q-X'_q)\left(I_q+\frac{E''_d}{x'_q-x''_q}-I_q-\frac{E'_d}{x'_q-x''_q}\right)}{T'_{qo}}$$

$$\frac{dE'_d}{dt} = \frac{-E'_d-\left(\frac{x_q-x'_q}{x'_q-x''_q}\right)E'_d+\left(\frac{x_q-x'_q}{x'_q-x''_q}\right)E''_d}{T'_{qo}}$$

$$\frac{dE'_d}{dt} = \frac{-\left(\frac{x_q-x''_q}{x'_q-x''_q}\right)E'_d+\left(\frac{x_q-x'_q}{x'_q-x''_q}\right)E''_d}{T'_{qo}}$$

The $\frac{d\psi'_q}{dt}$ and $\frac{d\psi'_d}{dt}$ are trivial. Thus in the end the derivative equations for GENROU without saturation with result in the GENTPF/GENTPJ dynamic equations without saturation as shown in the following table.

GENROU equation without saturation	Make assumptions $E_q'' = \psi_d'$; $E_d'' = \psi_q'$; $X_l = X_q''$ Gives you the GENTPF/GENTPJ equations
$\dfrac{d\psi_q'}{dt} = \dfrac{-\psi_q' + (X_q' - X_l)I_q + E_d'}{T_{qo}''}$	$\dfrac{dE_d''}{dt} = \dfrac{E_d' - E_d'' + (X_q' - X_l)I_q}{T_{qo}''}$
$\dfrac{d\psi_d'}{dt} = \dfrac{-\psi_d' - (X_d' - X_l)I_d + E_q'}{T_{do}''}$	$\dfrac{dE_q''}{dt} = \dfrac{E_q' - E_q'' - (X_d' - X_l)I_d}{T_{do}''}$
$\dfrac{dE_q'}{dt} = \dfrac{E_{fd} - \left(E_q' + (X_d - X_d')\left(I_d + \left(\dfrac{X_d' - X_d''}{(X_d' - X_l)^2}\right)(-\psi_d' - (X_d' - X_l)I_d + E_q')\right)\right)}{T_{do}'}$	$\dfrac{dE_q'}{dt} = \dfrac{E_{fd} - \left(\dfrac{X_d - X_d''}{X_d' - X_d''}\right)E_q' + \left(\dfrac{X_d - X_d'}{X_d' - X_d''}\right)E_q''}{T_{do}'}$ (See derivation following this table)
$\dfrac{dE_d'}{dt} = \dfrac{-E_d' + (X_q - X_q')\left(I_q - \dfrac{X_q' - X_q''}{(X_q' - X_l)^2}(-\psi_q' + (X_q' - X_l)I_q + E_d')\right)}{T_{qo}'}$	$\dfrac{dE_d'}{dt} = \dfrac{-\left(\dfrac{X_q - X_q''}{X_q' - X_q''}\right)E_d' + \left(\dfrac{X_q - X_q'}{X_q' - X_q''}\right)E_d''}{T_{qo}'}$ (See derivation following this table)

Taking these modified equations and applying saturation to the model will then be done more globally by multiplying all instances of E_q', E_d', E_q'' and E_d'' by the appropriate Sat_q or Sat_d function to represent the fact that the flux linkages become saturated. This then gives you the same derivate equations shown in the GENTPF/GENTPJ block diagrams.

	GENTPF/GENTPJ without Saturation	GENTPF/GENTPJ with Saturation
$\dfrac{dE_d''}{dt}$	$\dfrac{E_d' - E_d'' + (X_q' - X_l)I_q}{T_{qo}''}$	$\dfrac{Sat_d(E_d' - E_d'') + (X_q' - X_l)I_q}{T_{qo}''}$
$\dfrac{dE_q''}{dt}$	$\dfrac{E_q' - E_q'' - (X_d' - X_l)I_d}{T_{do}''}$	$\dfrac{Sat_d(E_q' - E_q'') - (X_d' - X_l)I_d}{T_{do}''}$
$\dfrac{dE_q'}{dt}$	$\dfrac{E_{fd} - \left(\dfrac{X_d - X_d''}{X_d' - X_d''}\right)E_q' + \left(\dfrac{X_d - X_d'}{X_d' - X_d''}\right)E_q''}{T_{do}'}$	$\dfrac{E_{fd} + Sat_q\left(-\left(\dfrac{X_d - X_d''}{X_d' - X_d''}\right)E_q' + \left(\dfrac{X_d - X_d'}{X_d' - X_d''}\right)E_q''\right)}{T_{do}'}$
$\dfrac{dE_d'}{dt}$	$\dfrac{-\left(\dfrac{X_q - X_q''}{X_q' - X_q''}\right)E_d' + \left(\dfrac{X_q - X_q'}{X_q' - X_q''}\right)E_d''}{T_{qo}'}$	$\dfrac{Sat_d\left(-\left(\dfrac{X_q - X_q''}{X_q' - X_q''}\right)E_d' + \left(\dfrac{X_q - X_q'}{X_q' - X_q''}\right)E_d''\right)}{T_{qo}'}$

3.3 K_{is} parameter in GENTPJ Model

The only difference between the GENTPF and GENTPJ model is the a parameter K_{is}. In the GENROU and GENSAL models the saturation is a function of only the air-gap flux ψ_{ag}. In generator testing however, it has been found that saturation is also a function of the armature current magnitude. The GENTPF model assumes that the saturation is NOT a function of armature current. The GENTPJ model recognizes that the leakage flux components induces in the stator teeth by high stator currents can increase the reluctance of the magnetic circuit. To model this, the argument of the saturation function is increased by a multiplication of the parameter K_{is} by the armature current and thus GENTPJ uses.

$$Sat_d = 1 + Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$$

$$Sat_q = 1 + \frac{X_q}{X_d}Sat\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$$