# Linear Analysis Techniques in PowerWorld Simulator

An overview of the underlying mathematics of the power flow and linearized analysis techniques

2001 South First Street
Champaign, Illinois 61820
+1 (217) 384.6330

support@powerworld.com
http://www.powerworld.com

# AC Power Flow Equations

- Full AC Power Flow Equations

$$P_k = 0 = V_k^2 g_{kk} + V_k \sum_{\substack{m=1 \\ m \neq k}}^{N} \left( V_m \left[ g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m) \right] \right) - P_{Gk} + P_{Lk}$$

$$Q_k = 0 = -V_k^2 b_{kk} + V_k \sum_{\substack{m=1 \\ m \neq k}}^{N} \left( V_m \left[ g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m) \right] \right) - Q_{Gk} + Q_{Lk}$$

- Solution requires iteration of equations

$$\begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\mathbf{V} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{P}}{\partial \boldsymbol{\delta}} & \frac{\partial \mathbf{P}}{\partial \mathbf{V}} \\ \frac{\partial \mathbf{Q}}{\partial \boldsymbol{\delta}} & \frac{\partial \mathbf{Q}}{\partial \mathbf{V}} \end{bmatrix}^{-1} \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix}$$

- Note: the large matrix (J) is called the Jacobian

# Full AC Derivatives

- Real Power derivative equations are

$$\frac{\partial P_k}{\partial \delta_m} = V_k V_m \left[-g_{km}\cos(\delta_k - \delta_m) - b_{km}\sin(\delta_k - \delta_m)\right] \qquad \frac{\partial P_k}{\partial V_m} = V_k \left[g_{km}\sin(\delta_k - \delta_m) - b_{km}\cos(\delta_k - \delta_m)\right]$$

$$\frac{\partial P_k}{\partial \delta_k} = V_k \sum_{\substack{m=1 \\ m \neq k}}^{N} \left[V_m \left[-g_{km}\sin(\delta_k - \delta_m) + b_{km}\cos(\delta_k - \delta_m)\right]\right] \qquad \frac{\partial P_k}{\partial V_k} = 2V_k\, g_{kk} + \sum_{\substack{m=1 \\ m \neq k}}^{N} \left[V_m \left[g_{km}\cos(\delta_k - \delta_m) + b_{km}\sin(\delta_k - \delta_m)\right]\right]$$

- Reactive Power derivative equations are

$$\frac{\partial Q_k}{\partial \delta_m} = V_k V_m \left[-g_{km}\cos(\delta_k - \delta_m) - b_{km}\sin(\delta_k - \delta_m)\right] \qquad \frac{\partial Q_k}{\partial V_m} = V_k \left[g_{km}\sin(\delta_k - \delta_m) - b_{km}\cos(\delta_k - \delta_m)\right]$$

$$\frac{\partial Q_k}{\partial \delta_k} = V_k \sum_{\substack{m=1 \\ m \neq k}}^{N} \left[V_m \left[g_{km}\cos(\delta_k - \delta_m) + b_{km}\sin(\delta_k - \delta_m)\right]\right] \qquad \frac{\partial Q_k}{\partial V_k} = -2V_k\, b_{kk} + \sum_{\substack{m=1 \\ m \neq k}}^{N} \left[V_m \left[g_{km}\sin(\delta_k - \delta_m) - b_{km}\cos(\delta_k - \delta_m)\right]\right]$$

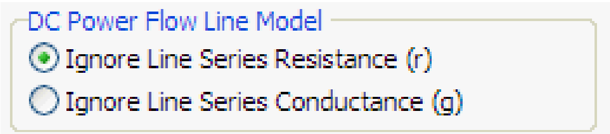
# Decoupled Power Flow Equations

- Make the following assumptions $\delta_k - \delta_m \approx 0 \quad \sin(\delta_k - \delta_m) \approx 0$
  $V_k \approx 1 \quad g_{km} \approx 0 \quad \cos(\delta_k - \delta_m) \approx 1$
- Derivatives simplify to

$$\frac{\partial P_k}{\partial \delta_k} = \sum_{\substack{m=1 \\ m \neq k}}^{N} b_{km} \qquad \frac{\partial Q_k}{\partial V_k} = -2b_{kk} + \sum_{\substack{m=1 \\ m \neq k}}^{N} (-b_{km}) \qquad \frac{\partial P_k}{\partial \delta_m} = -b_{km} \qquad \frac{\partial Q_k}{\partial V_m} = -b_{km}$$

$$\frac{\partial P_k}{\partial V_k} = 0 \qquad \frac{\partial P_k}{\partial V_m} = 0 \qquad \frac{\partial Q_k}{\partial \delta_k} = 0 \qquad \frac{\partial Q_k}{\partial \delta_m} = 0$$

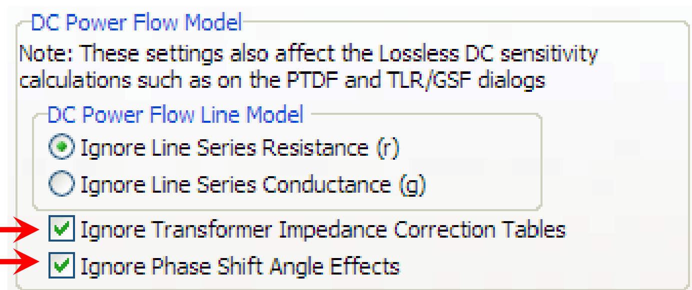
- Note: If assumption is instead $r_{km} \approx 0$ then replace all $b_{km}$ with $1/x_{km}$
- Option in Simulator Options under Power Flow Solution, DC Options sub-tab



# Other Typical Assumptions about Transformers

- Ignore Transformer Impedance Correction Tables
  - If we do not ignore, then as tap or phase changes the series branch impedances change
- Ignore Phase Shift Angle Effects
  - If we do not ignore, then as the phase angle changes the series impedance seen between the buses will vary
- Both of these are normally done because if they are not ignored, then the solution matrices become a function of the system state (i.e. impedance will vary with tap or phase)
  - This defeats the purposed of using the Decoupled equations.

Recommendation:
Leave these checked



# B’ and B’’ Matrices

- Define $\frac{\partial \mathbf{Q}}{\partial \mathbf{V}} = \mathbf{B}^{''}$ $\frac{\partial \mathbf{P}}{\partial \boldsymbol{\delta}} = \mathbf{B}^{'}$

- Now Iterate the “decoupled” equations

$$\Delta\boldsymbol{\delta} = \left[\mathbf{B}^{'}\right]^{-1} \Delta\mathbf{P}$$

$$\Delta\mathbf{V} = \left[\mathbf{B}^{''}\right]^{-1} \Delta\mathbf{Q}$$

- What are B’ and B’’?
  – B’ is the imaginary part of the Y-Bus with all the “shunt terms” removed
  – B’’ is the imaginary part of the Y-Bus with all the “shunt terms” double counted

# "DC Power Flow"

- The "DC Power Flow" equations are simply the real part of the decoupled power flow equations
  - Voltages and reactive power are ignored
  - Only angles and real power are solved for by iterating

$$\mathbf{\Delta\delta} = \left[\mathbf{B}^{'}\right]^{-1} \mathbf{\Delta P}$$

# Bus Voltage and Angle Sensitivities to a Transfer

- Power flow was solved by iterating $\begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\mathbf{V} \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix}$
- Model the transfer as a change in the injections $\Delta$P
  - Buyer: $\Delta\mathbf{T}_B = \begin{bmatrix} 0 & 0 & PF_{Bf} & 0 & PF_{Bg} & 0 \end{bmatrix}^T \quad \sum_{h=1}^{N} PF_{Bh} = 1$
  - Seller: $\Delta\mathbf{T}_S = \begin{bmatrix} 0 & PF_{Sx} & 0 & PF_{Sy} & 0 & 0 \end{bmatrix}^T \quad \sum_{z=1}^{N} PF_{Sz} = 1$
- Then solve for the voltage and angle sensitivities by solving $\begin{bmatrix} \Delta\boldsymbol{\delta}_S \\ \Delta\mathbf{V}_S \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta\mathbf{T}_S \\ \mathbf{0} \end{bmatrix}$ $\begin{bmatrix} \Delta\boldsymbol{\delta}_B \\ \Delta\mathbf{V}_B \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta\mathbf{T}_B \\ \mathbf{0} \end{bmatrix}$
- These are the sensitivities of the Buyer and Seller "sending power to the slack bus"

# What about Losses?

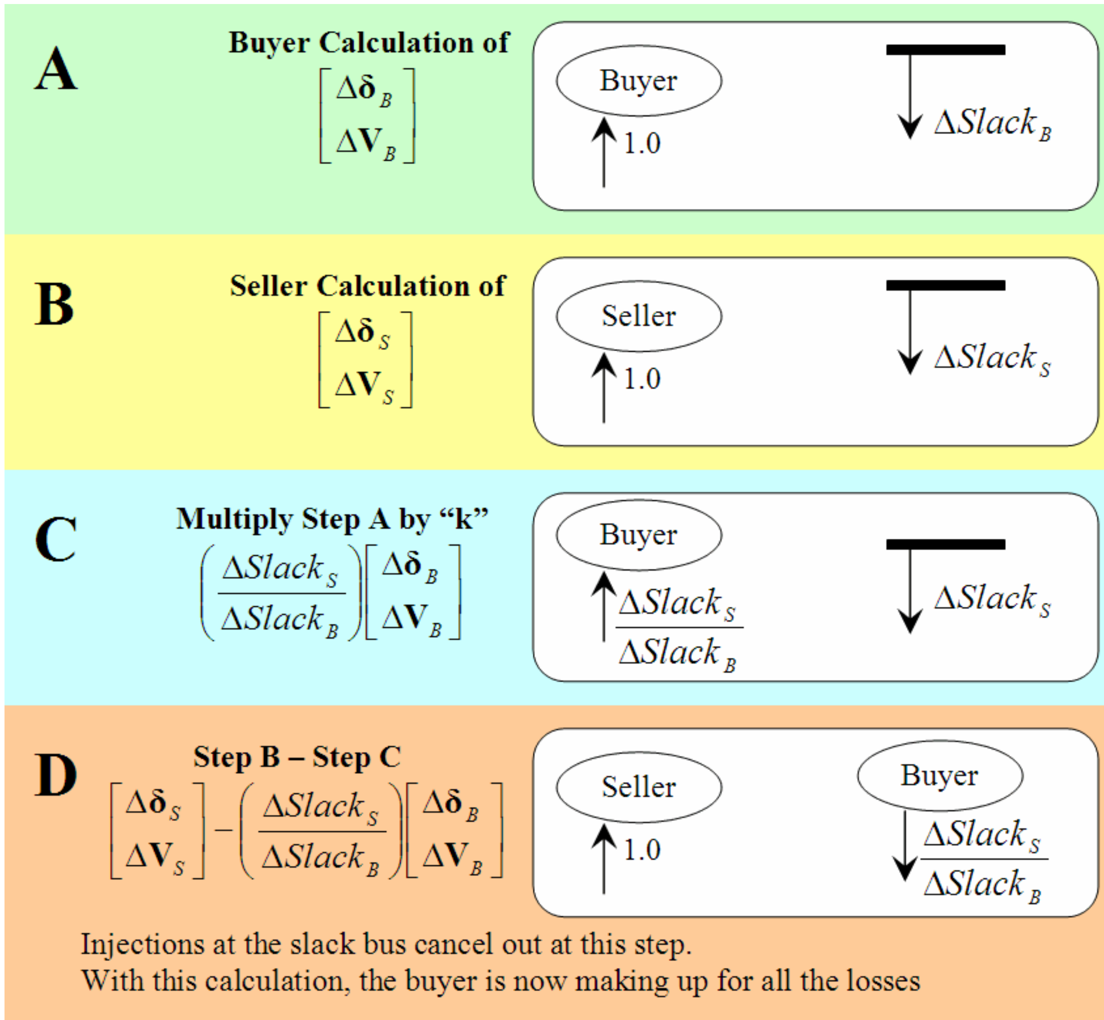
- If we assume the total sensitivity to the transfer is the seller minus the buyer sensitivity, then $\Delta\boldsymbol{\delta} = \Delta\boldsymbol{\delta}_S - \Delta\boldsymbol{\delta}_B$ and $\Delta\mathbf{V} = \Delta\mathbf{V}_S - \Delta\mathbf{V}_B$
- This makes assumption that ALL the change in losses shows up at the slack bus.
- Simulator assigns the change to the BUYER by defining

$$k = \frac{\Delta Slack_S}{\Delta Slack_B} = \frac{\text{Change in slack bus generation for seller sending power to slack}}{\text{Change in slack bus generation for buyer sending power to slack}}$$

- Then

$$\begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\mathbf{V} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{\delta}_S \\ \Delta\mathbf{V}_S \end{bmatrix} - k \begin{bmatrix} \Delta\boldsymbol{\delta}_B \\ \Delta\mathbf{V}_B \end{bmatrix}$$

# Why does “k” assign the losses to the Buyer?



# Lossless DC Voltage and Angle Sensitivities

- Use the DC Power Flow Equations

$$\Delta\boldsymbol{\delta} = \left[\mathbf{B}^{'}\right]^{-1} \Delta\mathbf{P}$$

- Then determine angle sensitivities

$$\Delta\boldsymbol{\delta}_S = \left[\mathbf{B}^{'}\right]^{-1} \Delta\mathbf{T}_S \qquad \Delta\boldsymbol{\delta}_B = \left[\mathbf{B}^{'}\right]^{-1} \Delta\mathbf{T}_B$$

- The DC Power Flow ignores losses, thus

$$\Delta\boldsymbol{\delta} = \Delta\boldsymbol{\delta}_S - \Delta\boldsymbol{\delta}_B$$

# Lossless DC Sensitivities with Phase Shifters Included

- DC Power Flow equations $\left[\mathbf{B}^{'}\right]\Delta\boldsymbol{\delta} = \Delta\mathbf{P}$
- Augmented to include an equation that describes the change in flow on a phase-shifter controlled branch as being zero.

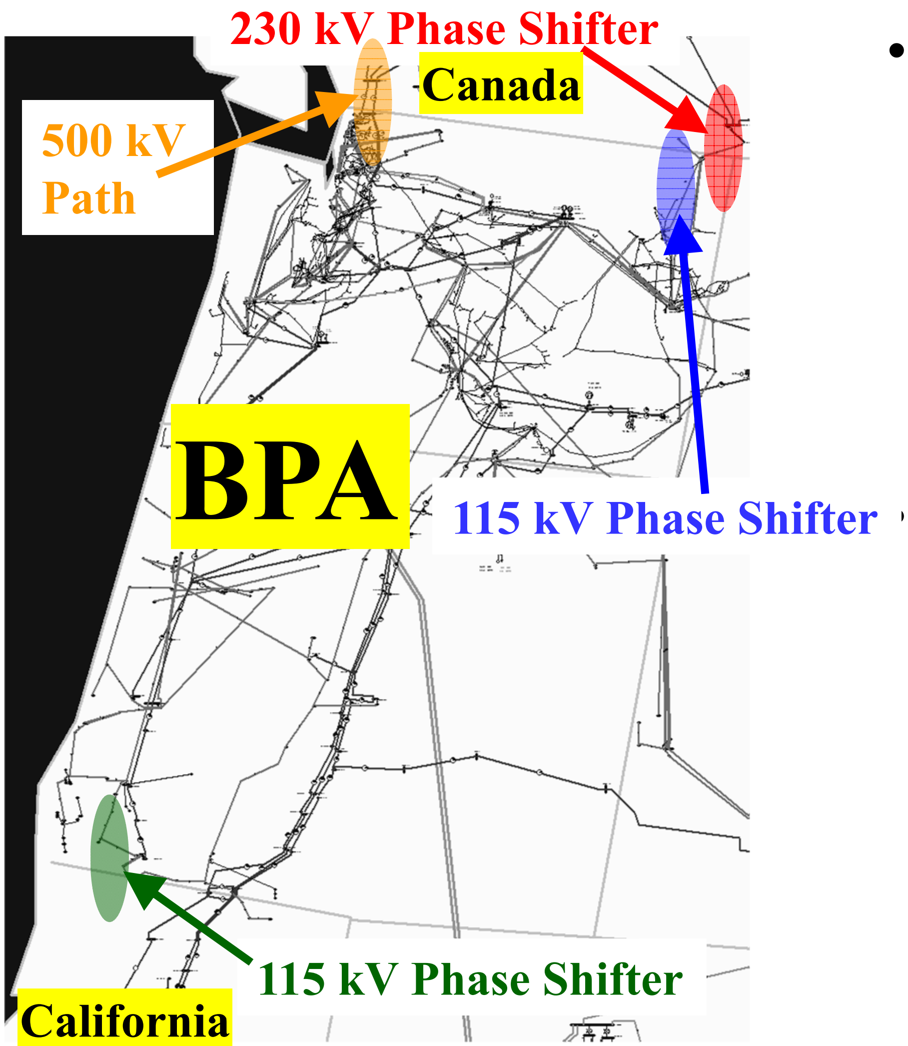
$$\text{Line Flow Change} = \mathbf{B}_{\delta}\Delta\boldsymbol{\delta} + \mathbf{B}_{\alpha}\Delta\boldsymbol{\alpha} = \mathbf{0}$$

- Thus instead of DC power flow equations we use

$$\begin{bmatrix}\Delta\boldsymbol{\delta}\\ \Delta\boldsymbol{\alpha}\end{bmatrix} = \begin{bmatrix}\mathbf{B}^{'} & \mathbf{0}\\ \mathbf{B}_{\delta} & \mathbf{B}_{\alpha}\end{bmatrix}^{-1}\begin{bmatrix}\Delta\mathbf{P}\\ \mathbf{0}\end{bmatrix}$$

- Otherwise process is the same.

# Lossless DC with Phase Shifters




- Phase Shifters are often on lower voltage paths (230 kV or less) with relatively small limits
  - They are put there in order to manage/prevent the flow on a path that would commonly see overloads
  - Thus, they constantly show up as "overloaded" when using linear analysis if they are not accounted for
- Example: Border of Canada with Northwestern United States
  - PTDFs between Canada and US without Phase-Shifters
    - 85% on 500 kV Path
    - 15% on Eastern Path
  - PTDF With Phase-Shifters
    - 100% goes on 500 kV Path
    - 0% on Eastern Path
    - This better reflects real system

# Power Transfer Distribution Factors (PTDFs)

- PTDF: measures the sensitivity of line MW flows to a MW transfer.
  - Line flows are simply a function of the voltages and angles at its terminal buses
  - Thus, the PTDF is simply a function of these voltage and angle sensitivities.
    - This is the "Chain Rule" from Calculus
- $P_{km}$ is the flow from bus k to bus m

$$PTDF = \Delta P_{km} = \left[\frac{\partial P_{km}}{\partial V_k}\right]\Delta V_K + \left[\frac{\partial P_{km}}{\partial V_m}\right]\Delta V_m + \left[\frac{\partial P_{km}}{\partial \delta_k}\right]\Delta \delta_K + \left[\frac{\partial P_{km}}{\partial \delta_m}\right]\Delta \delta_m$$

**Voltage and Angle Sensitivities that were just determined**

# $P_{km}$ Derivative Calculations

- Full AC equations

$$\left[\frac{\partial P_{km}}{\partial \delta_m}\right] = V_k V_m\left[g_{km}\sin\left(\delta_k - \delta_m\right) - b_{km}\cos\left(\delta_k - \delta_m\right)\right]$$

$$\left[\frac{\partial P_{km}}{\partial \delta_k}\right] = V_k V_m\left[-g_{km}\sin\left(\delta_k - \delta_m\right) + b_{km}\cos\left(\delta_k - \delta_m\right)\right]$$

$$\left[\frac{\partial P_{km}}{\partial V_m}\right] = V_k\left[g_{km}\cos\left(\delta_k - \delta_m\right) + b_{km}\sin\left(\delta_k - \delta_m\right)\right]$$

$$\left[\frac{\partial P_{km}}{\partial V_k}\right] = 2V_k g_{kk} + V_m\left[g_{km}\cos\left(\delta_k - \delta_m\right) + b_{km}\sin\left(\delta_k - \delta_m\right)\right]$$

- Lossless DC Approximations yields

$$\left[\frac{\partial P_{km}}{\partial \delta_m}\right] = -b_{km} \qquad \left[\frac{\partial P_{km}}{\partial \delta_k}\right] = b_{km}$$

$$\left[\frac{\partial P_{km}}{\partial V_k}\right] = 0 \qquad \left[\frac{\partial P_{km}}{\partial V_m}\right] = 0$$

# What do Flow Sensitivities (PTDFs, GSFs, TLRs, …) give us?

- Give the ability to model a change in power injection without actually doing a new power flow solution
  - Transfer of power between two places
  - Generator outage
  - Load outage
- Still can't model a line outage or line closure yet, but that is what LODFs will give us

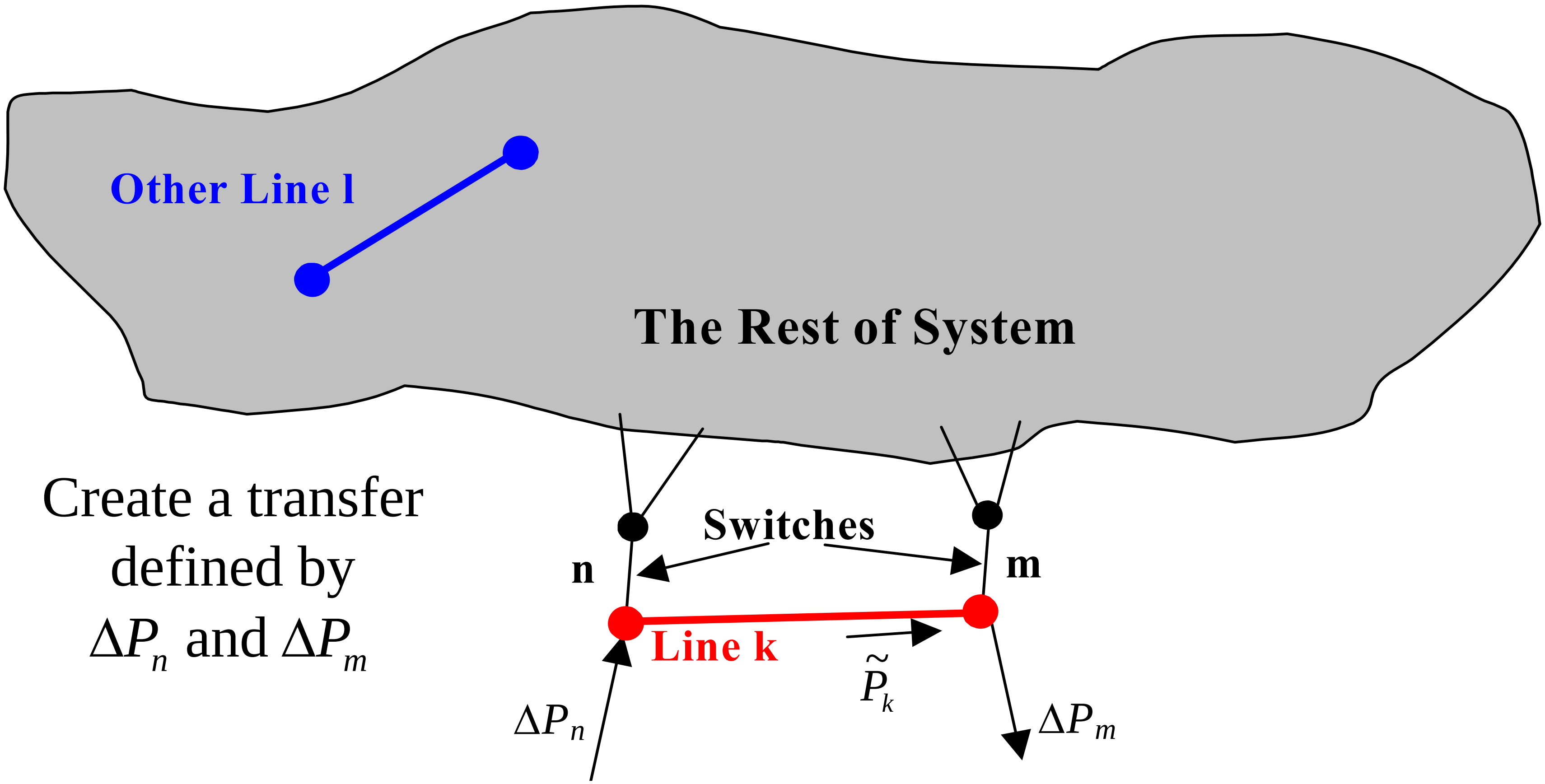
# Line Outage Distribution Factors (LODFs)

- $LODF_{l,k}$: percent of the pre-outage flow on Line K will show up on Line L after the outage of Line K

$$LODF_{l,k} = \frac{\Delta P_{l,k}}{P_k}$$

- Linear impact of an outage is determined by modeling the outage as a "transfer" between the terminals of the line

# Modeling an LODF as a Transfer




Create a transfer defined by $\Delta P_n$ and $\Delta P_m$

Assume $\tilde{P}_k = \Delta P_n = \Delta P_m$
Then the flow on the Switches is ZERO, thus Opening Line K is equivalent to the “transfer”

# Modeling an LODF as a Transfer

- Thus, setting up a transfer of $\tilde{P}_k$ MW from Bus n to Bus m is equivalent to outaging the transmission line
- Let's assume we know what $\tilde{P}_k$ is equal to, then we can calculate the values relevant to the LODF.

# Calculation of LODF

- Estimate of post-outage flow on Line L

$$\Delta P_{l,k} = PTDF_l * \tilde{P}_k$$

- Estimate of flow on Line L after transfer

$$\tilde{P}_k = P_k + PTDF_k * \tilde{P}_k \longrightarrow \tilde{P}_k = \frac{P_k}{1 - PTDF_k}$$

- Thus we can write

$$LODF_{l,k} = \frac{\Delta P_{l,k}}{P_k} = \frac{PTDF_l * \tilde{P}_k}{P_k} = \frac{PTDF_l * \left(\frac{P_k}{1 - PTDF_k}\right)}{P_k}$$

$$\longrightarrow LODF_{l,k} = \frac{PTDF_l}{1 - PTDF_k}$$

- We have a simple function of PTDF values

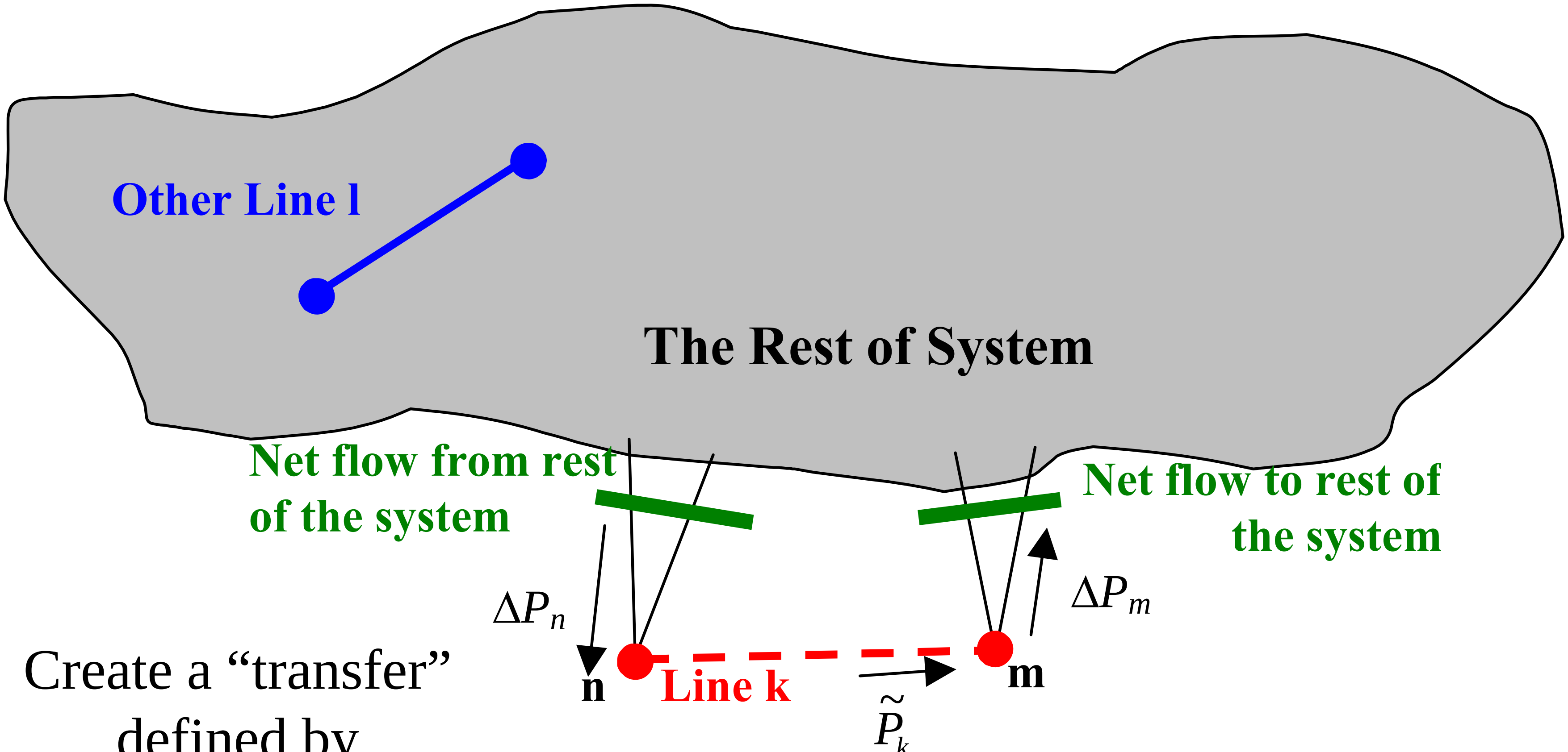
# Line Closure Distribution Factors (LCDFs)

- $LCDF_{l,k}$: percent of the post-closure flow on Line K will show up on Line L after the closure of Line K

$$LCDF_{l,k} = \frac{\Delta P_{l,k}}{\tilde{P}_k}$$

- Linear impact of an closure is determined by modeling the closure as a “transfer” between the terminals of the line

# Modeling the LCDF as a Transfer




Create a "transfer" defined by $\Delta P_n$ and $\Delta P_m$

Assume $\tilde{P}_k = \Delta P_n = \Delta P_m$
Then the net flow to and from the rest of the system are both zero, thus closing line k is equivalent the "transfer"

# Modeling an LCDF as a Transfer

- Thus, setting up a transfer of $-\tilde{P}_k$ MW from Bus n to Bus m is linearly equivalent to outaging the transmission line
- Let's assume we know what $-\tilde{P}_k$ is equal to, then we can calculate the values relevant to the LODF.
- Note: The negative sign is used so that the notation is consistent with LODF "transfer"

# Calculation of LCDF

- Estimate of post-closure flow on Line L

$$\Delta P_{l,k} = -PTDF_l * \tilde{P}_k$$

- Thus we can write

$$LCDF_{l,k} = \frac{\Delta P_{l,k}}{\tilde{P}_k} = \frac{-PTDF_l * \tilde{P}_k}{\tilde{P}_k} = -PTDF_l$$

$$\longrightarrow \quad LCDF_{l,k} = -PTDF_l$$

- Thus the *LCDF,* is exactly equal to the PTDF for a transfer between the terminals of the line

# LODF and LCDF

- LODF (LCDF) – Gives the ability to model a single line outage (closure) event
  - OTDF – incremental impact on a particular branch while also considering a single line outage
  - OMW – estimate of the flow on a particular branch after a single line outage

$$OTDF_{M,1} = PTDF_M + LODF_{M,1} * PTDF_1$$

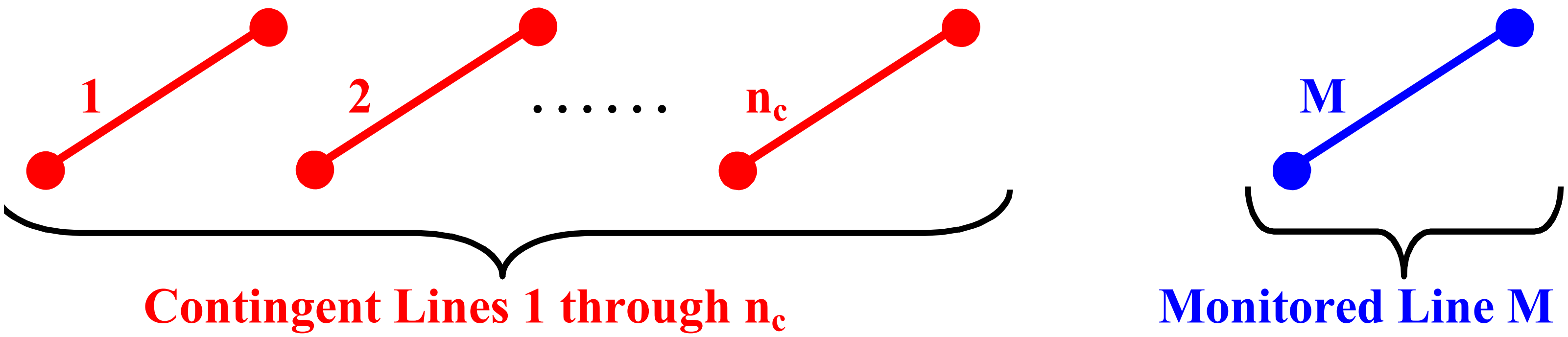
$$OMW_{M,1} = MW_M + LODF_{M,1} * MW_1$$

- Still can't model multiple line outages simultaneously
  - Can not just sum because the lines that are being outaged also interact with each other

$$OTDF_{M,K} = PTDF_M + \sum_{k=lineoutages} LODF_{M,k} * PTDF_k$$

$$OMW_{M,K} = MW_M + \sum_{k=lineoutages} LODF_{M,k} * MW_k$$

# Linear Impact of a Contingency with Multiple Outages




- Outage Transfer Distribution Factors (OTDFs)
  – The percent of a transfer that will flow on a branch M after the contingency occurs
- Outage Flows (OMWs)
  – The estimated flow on a branch M after the contingency occurs

# OTDFs and OMWs

- Single Contingency

$$OTDF_{M,1} = PTDF_M + LODF_{M,1} * PTDF_1$$

$$OMW_{M,1} = MW_M + LODF_{M,1} * MW_1$$

- Multiple Contingencies

$$OTDF_{M,C} = PTDF_M + \sum_{K=1}^{n_C} \left[ LODF_{MK} * NetPTDF_K \right]$$

$$OMW_{M,C} = MW_M + \sum_{K=1}^{n_C} \left[ LODF_{MK} * NetMW_K \right]$$

- What are $NetPTDF_K$ and $NetMW_K$?

# Determining *NetPTDF*$_K$ and *NetMW*$_K$

- Each *NetPTDF*$_K$ is a function of all the other *NetPTDFs* because the change in status of a line affects all other lines.
- Assume we know all *NetPTDFs* except for *NetPTDF*$_1$. Then we can write:

$$NetPTDF_1 = PTDF_1 + LODF_{12} NetPTDF_2 + \ldots + LODF_{1n_C} NetPTDF_{n_C}$$

$$= PTDF_1 + \sum_{K=2}^{n_C} \left[ LODF_{1K} NetPTDF_K \right]$$

- In general for each Contingent Line N, write

$$NetPTDF_N - \sum_{\substack{K=1 \\ K \neq N}}^{n_C} \left[ LODF_{NK} NetPTDF_K \right] = PTDF_N$$

# Determining $NetPTDF_K$ and $NetMW_K$

- Thus we have a set of $n_c$ equations and $n_c$ unknowns ($n_c$= number of contingent lines)

Known Values

$$\begin{bmatrix} 1 & -LODF_{12} & -LODF_{13} & \cdots & -LODF_{1n_C} \\ -LODF_{21} & 1 & -LODF_{23} & \cdots & -LODF_{2n_C} \\ -LODF_{31} & -LODF_{32} & 1 & \cdots & -LODF_{3n_C} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -LODF_{n_C1} & -LODF_{n_C2} & -LODF_{n_C3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} NetPTDF_1 \\ NetPTDF_2 \\ NetPTDF_3 \\ \vdots \\ NetPTDF_{n_C} \end{bmatrix} = \begin{bmatrix} PTDF_1 \\ PTDF_2 \\ PTDF_3 \\ \vdots \\ PTDF_{n_C} \end{bmatrix}$$

- Thus $\mathbf{NetPTDF_C} = [\mathbf{LODF_{CC}}]^{-1}\mathbf{PTDF_C}$
- Same type of derivation shows

$$\mathbf{NetMW_C} = [\mathbf{LODF_{CC}}]^{-1}\mathbf{MW_C}$$

# Operating and Limiting Circles

- Operating circle defines a circle of valid MW and Mvar values for a transmission line as a transfer takes place across the system
- Limiting circle has a radius equal to the MVA limit of the line
- The operating circle is utilized when using one of the DC methods and modeling reactive power by assuming constant voltage magnitudes
- Contingency analysis looks up the Mvar value corresponding to the calculated MW value
- ATC analysis and DC power flow find the intersection of the operating circle and the limiting circle to assign adjusted limits to lines

# Operating and Limiting Circles